Under what conditions do quantum systems thermalize?

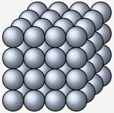
Christian Gogolin, Markus Müller, Arnau Riera, and Jens Eisert

Dahlem Center for Complex Quantum Systems, Freie Universität Berlin

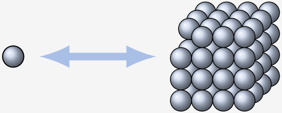
Institute for Physics and Astronomy, Universität Potsdam

Workshop “Information and Foundations of Thermodynamics”
at ETH Zürich 2011

Understanding thermalization



Understanding thermalization



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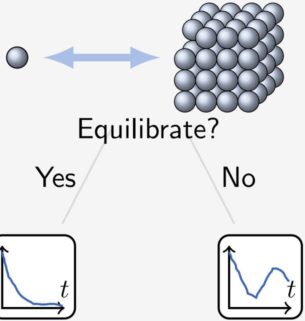

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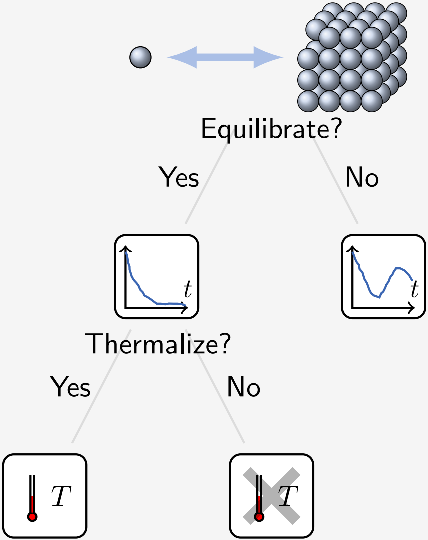

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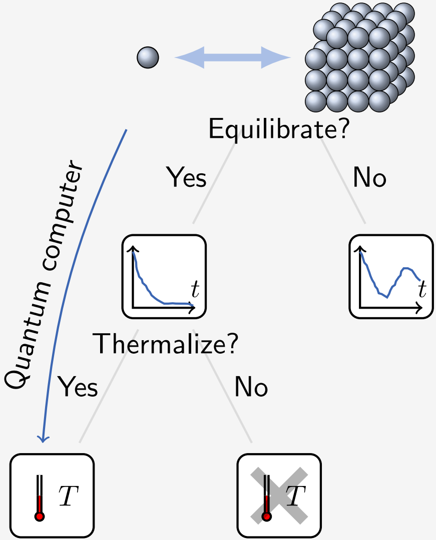

Setup

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System, $\mathcal{H}_S, \mathscr{H}_S$

$d_S = \dim(\mathcal{H}_S)$

Bath, $\mathcal{H}_B, \mathscr{H}_B$

$d_B \gg d_S$

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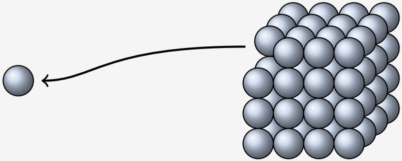

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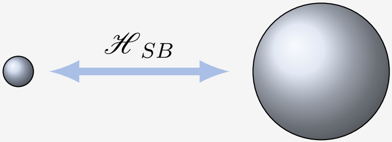

$$\mathscr{H} = \mathscr{H}_S \otimes \mathbb{1} + \mathbb{1} \otimes \mathscr{H}_B + \mathscr{H}_{SB}$$

$$\frac{d\psi_t}{dt} = \mathrm{i}\,[\psi_t, \mathscr{H}]$$

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System, $\mathcal{H}_S, \mathscr{H}_S$

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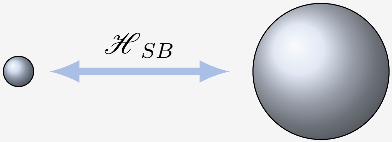

$$\psi_t^S = \mathrm{Tr}_B[\psi_t]$$

$$\mathscr{H} = \mathscr{H}_S \otimes \mathbb{1} + \mathbb{1} \otimes \mathscr{H}_B + \mathscr{H}_{SB} \qquad \frac{d\psi_t}{dt} = \mathrm{i}\,[\psi_t, \mathscr{H}]$$

Equilibration

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Theorem 1 (Equilibration [2])

If $\mathscr{H}$ has non-degenerate energy gaps, then for every $\psi_0 = |\psi_0\rangle\langle\psi_0|$ there exists a ω^S such that:

$$\overline{\mathcal{D}(\psi_t^S, \omega^S)} \leq \frac{1}{2}\sqrt{\frac{d_S^2}{d^{\text{eff}}}}.$$

[1] P. Reimann, PRL 101, 190403 (2008)
[2] N. Linden, S. Popescu, A. J. Short, and A. Winter, PRE 79, 061103 (2009)

Equilibration

Theorem

If $\mathscr{H}$ *has*
there exis

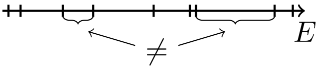
$\langle\psi_0|$

Non-degenerate energy gaps

$\mathscr{H}$ has non–degenerate energy gaps iff:

$$E_k - E_l = E_m - E_n$$

$$\implies k = l \wedge m = n \quad \vee \quad k = m \wedge l = n$$



Intuition: Sufficient for $\mathscr{H}$ to be fully interactive

$$\mathscr{H} \neq \mathscr{H}_1 \otimes \mathbb{1} + \mathbb{1} \otimes \mathscr{H}_2$$

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Effective dimension

$$d^{\text{eff}} = \frac{1}{\sum_k |\langle E_k|\psi_0\rangle|^4}.$$

Intuition: Dimension of supporting energy subspace

[1] P. Reimann, PRL 101, 190403 (2008)
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$$\overline{\mathcal{D}(\psi_t^S, \omega^S)} \leq \frac{1}{2}\sqrt{\frac{d_S^2}{d^{\mathrm{eff}}}}.$$

$\Longrightarrow$ If $d^{\mathrm{eff}} \gg d_S^2$ then ψ_t^S equilibrates.

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Maximum entropy principle

Theorem 2 (Maximum entropy principle [3])

If $\mathrm{Tr}[A\,\psi_t]$ *equilibrates, it equilibrates towards its time average*

$$\overline{\mathrm{Tr}[A\,\psi_t]} = \mathrm{Tr}[A\,\overline{\psi_t}] = \mathrm{Tr}[A\,\omega],$$

$$\text{where } \omega = \sum_k \pi_k \psi_0 \pi_k$$

(with π_k *the energy eigen projectors) is the dephased state that maximizes the von Neumann entropy, given all conserved quantities.*

[3] C. Gogolin, M. P. Müller, and J. Eisert, PRL 106, 040401 (2011)
[4] M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, PRL 98, 050405 (2007)

Maximu

Theorem

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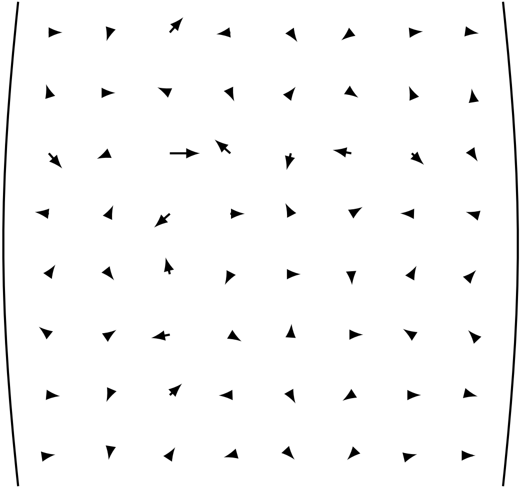
Time averaging

$\psi_0 =$ 

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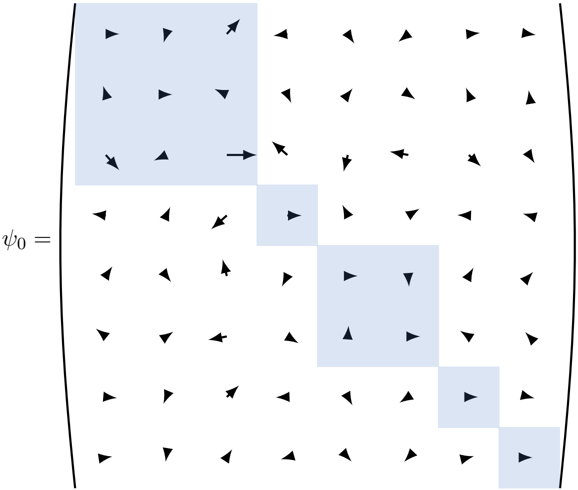

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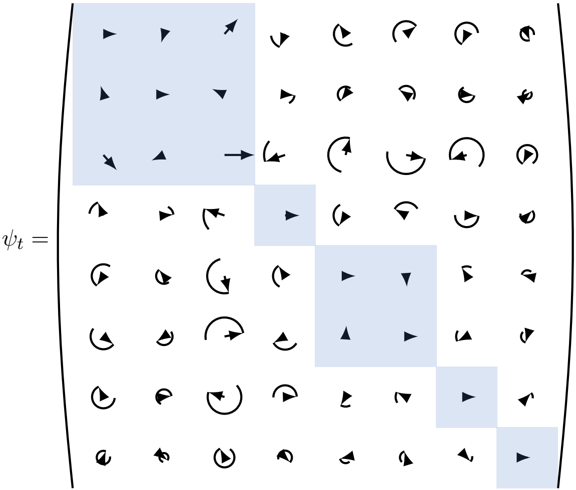

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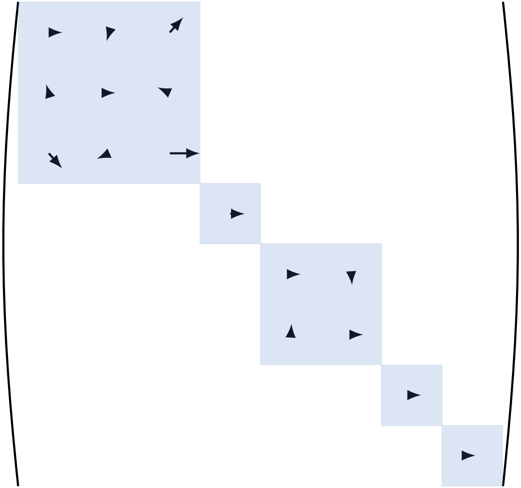

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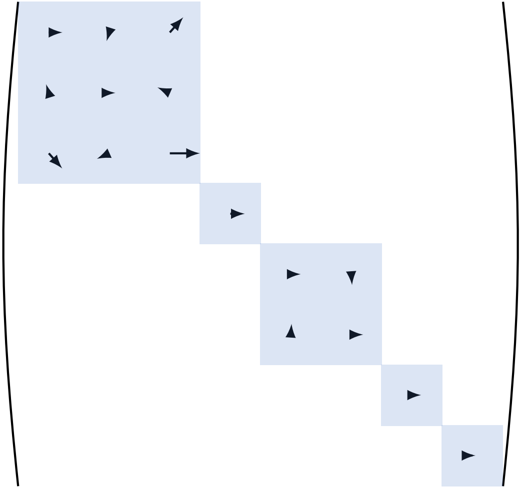
$\omega =$ 

$\psi_0 \to \omega$ is a pinching $\Rightarrow$ ω maximizes entropy.

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(with π_k *the energy eigen projectors) is the dephased state that maximizes the von Neumann entropy, given all conserved quantities.*

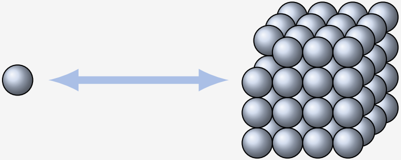
$\Rightarrow$ Maximum entropy principle from pure quantum dynamics.

Proves a conjecture from the condensed matter literature from 2007 [4].

[3] C. Gogolin, M. P. Müller, and J. Eisert, PRL 106, 040401 (2011)
[4] M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, PRL 98, 050405 (2007)

Thermalization and realistic weak coupling

Thermalization is a complicated process



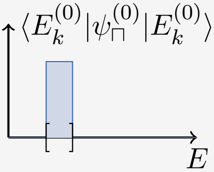
Thermalization implies:

1. Equilibration [1, 2, 7]
2. Subsystem initial state independence [3]
3. Weak bath state dependence [5]
4. Diagonal form of the subsystem equilibrium state [6]
5. Gibbs state $\omega^S = \mathrm{Tr}_B[\omega] \approx \mathrm{e}^{-\beta \mathscr{H}_S}$ [5]

[1] P. Reimann, PRL 101, 190403 (2008)
[2] N. Linden, S. Popescu, A. J. Short, and A. Winter, PRE 79, 061103 (2009)
[3] C. Gogolin, M. P. Müller, and J. Eisert, PRL 106, 040401 (2011)
[5] A. Riera, C. Gogolin, and J. Eisert, 1102.2389
[6] C. Gogolin, PRE 81, 051127 (2010)
[7] J. Gemmer, M. Michel, and G. Mahler, Springer (2009)

Level counting with no coupling

$$\| \mathscr{H}_{SB} \|_\infty = 0$$



[8] S. Goldstein, PRL 96, 050403 (2006)

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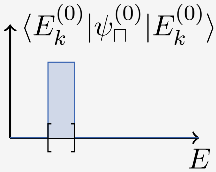

Well known fact [8]:

no coupling ↓ # bath states in $[E - E_k^S, E - E_k^S + \Delta]$

$$\omega_\sqcap^{S(0)} \propto \sum_k \Omega_\Delta^B(E - E_k^S)\, |E_k^S\rangle\langle E_k^S|$$

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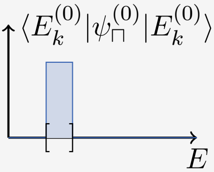

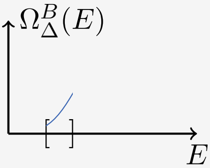

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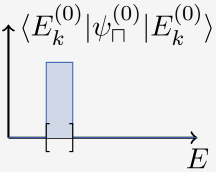

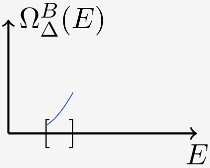

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↑ exponentially dense spectrum

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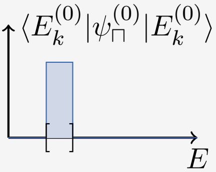

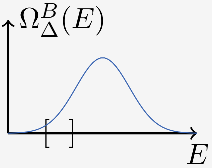

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Perturbative coupling $\| \mathscr{H}_{SB} \|_\infty < \mathrm{gaps}(\mathscr{H}_0)$ ...

- ... is unrealistic as the spectrum of $\mathscr{H}_0$ becomes exponentially dense.

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perturbative coupling

$\Downarrow$ [3]

effective entanglement in the eigenbasis $R(\psi_0)$ is small

$\Downarrow$ [3]

absence of initial state independence.

$$\mathcal{D}(\omega^{S(1)}, \omega^{S(2)}) \geq \mathcal{D}(\psi_0^{S(1)}, \psi_0^{S(2)}) - R(\psi_0^{S(1)}) - R(\psi_0^{S(2)})$$

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Effective entanglement in the eigenbasis

$$R(\psi_0) = \sum_k |\langle E_k|\psi_0\rangle|^2 \, \mathcal{D}(\mathrm{Tr}_B \, |E_k\rangle\langle E_k|, \psi_0^S)$$

Measures how entangled the eigenbasis feels for the given initial state.

all

$$\mathcal{D}(\omega^{S(1)}, \omega^{S(2)}) \geq \mathcal{D}(\psi_0^{S(1)}, \psi_0^{S(2)}) - R(\psi_0^{S(1)}) - R(\psi_0^{S(2)})$$

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- . . . pr…

… all

$S^{(2)})$

Theorem 3 (Entanglement in eigenbasis)

For every orthonormal basis $\{|i\rangle\}$ for S and every initial product state with $\psi_0 = |j\rangle\langle j| \otimes \phi_0^B$, the effective entanglement in the eigenbasis (for non-degenerate $\mathscr{H}$) is on average upper bounded by

$$\mathbb{E}_{\phi_0^B} R(|j\rangle\langle j| \otimes \phi_0^B) \leq 2\,\delta\,d_S,$$

where

$$\delta = \max_k \min_i \mathcal{D}(\mathrm{Tr}_B\,|E_k\rangle\langle E_k|, |i\rangle\langle i|)$$

is the geometric measure of entanglement of the eigenstate $|E_k\rangle$ with respect to the basis $\{|i\rangle\}$.

[3] C. Gogolin, M. P. Müller, and J. Eisert, PRL 106, 040401 (2011)

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$\Longrightarrow$ Refutes wide spread believe that "non-integrable models thermalize."

[3] C. Gogolin, M. P. Müller, and J. Eisert, PRL 106, 040401 (2011)

Realistic weak coupling

- Naive perturbation theory fails.
- More realistic weak coupling:

$$\mathrm{gaps}(\mathscr{H}_0) \ll \| \mathscr{H}_{SB} \|_\infty \ll \Delta$$

[5] A. Riera, C. Gogolin, and J. Eisert, 1102.2389

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Theorem 4 (Corollary of a theorem from [5])

If $\| \mathscr{H}_{SB} \|_\infty \ll \Delta$ the dephased states $\omega_\sqcap^{S(0)}$ and $\omega_\sqcap^{S}$ are close to each other in the sense that

$$\mathcal{D}(\omega_\sqcap^{S}, \omega_\sqcap^{S(0)}) \lessapprox 3\sqrt{\frac{\| \mathscr{H}_{SB} \|_\infty}{2\Delta}}.$$

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$$\mathcal{D}(\omega_\sqcap^{S}, \omega_\sqcap^{S(0)}) \lessapprox \frac{\sqrt{\beta \| \mathscr{H}_{SB} \|_\infty}}{1 - \mathrm{e}^{-\beta\Delta}}.$$

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[5] A. Riera, C. Gogolin, and J. Eisert, 1102.2389

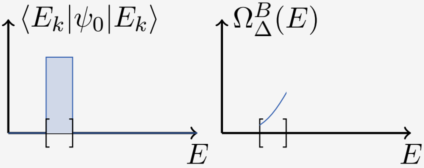
Putting everything together

Consequences

[5] A. Riera, C. Gogolin, and J. Eisert, 1102.2389

Consequences

$$\| \mathscr{H}_{SB} \|_\infty \ll \text{gaps}(\mathscr{H}_0)$$



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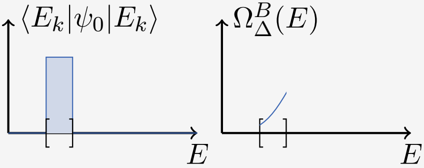

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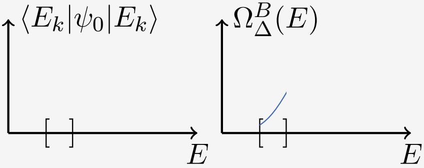

[5] A. Riera, C. Gogolin, and J. Eisert, 1102.2389

Consequences

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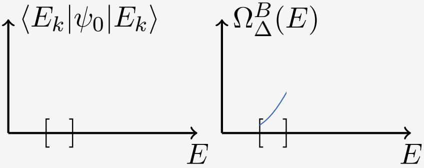

⟹ "Theorem" 5 (Theorem 2 in [5])

(Kinematic) Almost all pure states from a microcanonical subspace $[E, E + \Delta]$ are locally close to a Gibbs state.

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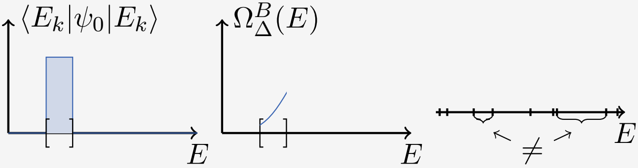

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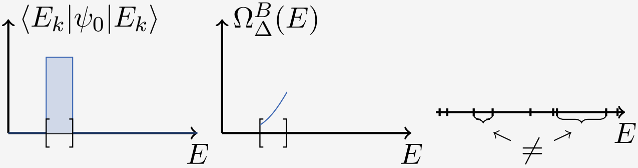

$\Longrightarrow$ "Theorem" 5 (Theorem 2 in [5])

(Kinematic) Almost all pure states from a microcanonical subspace $[E, E + \Delta]$ are locally close to a Gibbs state.

(Dynamic) All initial states $\psi_{\sqcap,0}$ locally equilibrate towards a Gibbs state, even if they are initially far from equilibrium.

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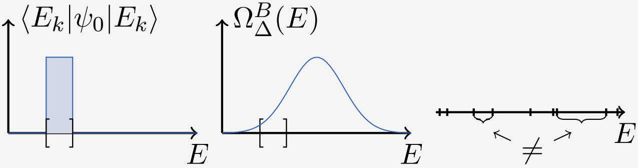

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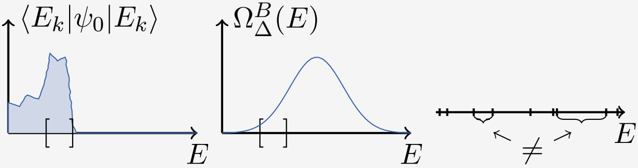

$\implies$ "Theorem" 5 (Theorem 2 in [5])

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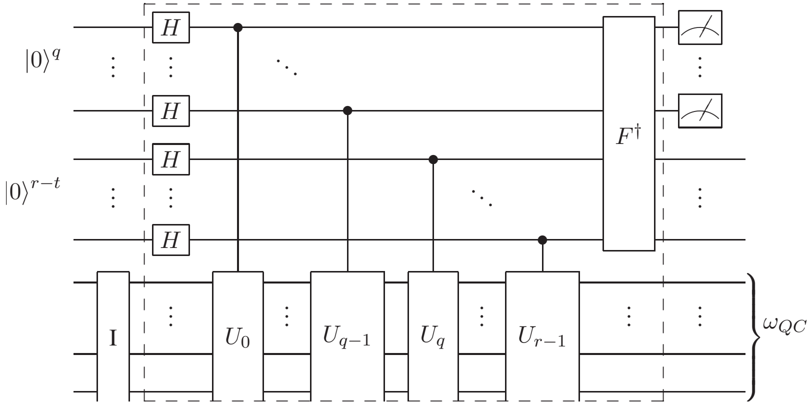
A quantum algorithm for Gibbs state preparation

A quantum algorithm for Gibbs state preparation

Quantum circuit



[5] A. Riera, C. Gogolin, and J. Eisert, 1102.2389
[9] D. Poulin and P. Wocjan, PRL 103, 220502 (2009)

A quantum algorithm for Gibbs state preparation

Quantum circuit

- No detailed knowledge about $\mathscr{H}_S$ is required.

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[9] D. Poulin and P. Wocjan, PRL 103, 220502 (2009)

A quantum algorithm for Gibbs state preparation

Quantum circuit

$|0\rangle^{q}$

$|0\rangle^{r-t}$

- No detailed knowledge about $\mathscr{H}_S$ is required.
- The algorithm uses partial phase estimation to prepare $\omega_{\sqcap}$.

ω_{QC}

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A quantum algorithm for Gibbs state preparation

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- No detailed knowledge about $\mathscr{H}_S$ is required.
- The algorithm uses partial phase estimation to prepare $\omega_{\sqcap}$.
- Complements quantum Metropolis
  - Trace distance error bound
  - Explicit runtime

(also compare [9])

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- The algorithm uses partial phase estimation to prepare $\omega_{\sqcap}$.
- Complements quantum Metropolis
  - Trace distance error bound
  - Explicit runtime

  (also compare [9])
- Complexity for fixed trace distance error:
  - polynomially many ancilla qubits
  - exponential runtime

  ($\Omega(\mathrm{e}^{\beta(E_2^S - E_1^S)})$ is necessary)

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[9] D. Poulin and P. Wocjan, PRL 103, 220502 (2009)

Collaborators

Freie Universität Berlin

Arnau Riera

Martin Kliesch

Jens Eisert

PERIMETER INSTITUTE

Markus P. Müller

University of BRISTOL

Andreas Winter

References

Thank you for your attention!

⟶ slides: www.cgogolin.de

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