What it takes to shun equilibration

Christian Gogolin

ICFO - The Institute of Photonic Sciences

Wallenberg Research Centre Stellenbosh
2018-03-12

Joint work with:
R. Gallego, H. Wilming, and J. Eisert

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A short digression into long-range systems. . .

PRL **119,** 110601 (2017) PHYSICAL REVIEW LETTERS week ending 15 SEPTEMBER 2017

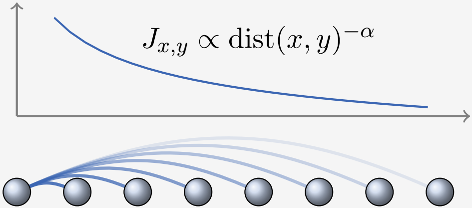
Correlation Decay in Fermionic Lattice Systems with Power-Law Interactions at Nonzero Temperature

Senaida Hernández-Santana,[1] Christian Gogolin,[1,2] J. Ignacio Cirac,[2] and Antonio Acín[1,3]
[1]*ICFO-Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain*
[2]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany*
[3]*ICREA-Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

(Received 22 March 2017; published 13 September 2017)

We study correlations in fermionic lattice systems with long-range interactions in thermal equilibrium. We prove a bound on the correlation decay between anticommuting operators and generalize a long-range Lieb-Robinson-type bound. Our results show that in these systems of spatial dimension D with, not necessarily translation invariant, two-site interactions decaying algebraically with the distance with an exponent $\alpha \geq 2D$, correlations between such operators decay at least algebraically to 0 with an exponent arbitrarily close to α at any nonzero temperature. Our bound is asymptotically tight, which we demonstrate

Correlation decay for power-law Hamiltonians (Fermions)



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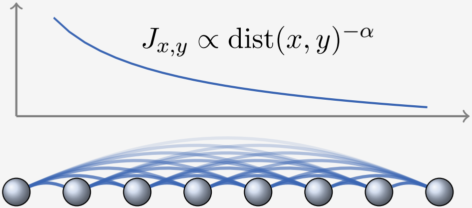

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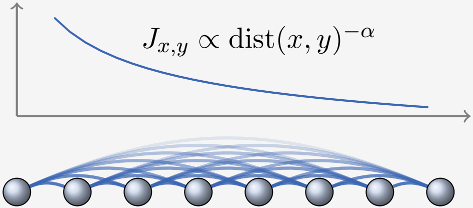

$$\mathrm{corr}(A,B)_\beta := \langle A\,B\rangle_\beta - \langle A\rangle_\beta\,\langle B\rangle_\beta,$$

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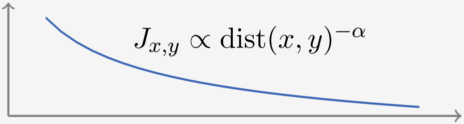

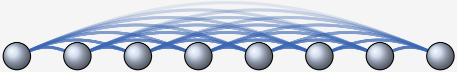

$$\mathrm{corr}(A,B)_\beta := \langle A\,B\rangle_\beta - \langle A\rangle_\beta \, \langle B\rangle_\beta,$$

Theorem (Correlation decay for long-range Hamiltonians [1])

For any $\alpha > 2\,D$ two-site power-law Hamiltonian on a D-dimensional square lattice and any odd operators A, B and temperature $T > 0$

$$|\,\mathrm{corr}(A,B)_\beta| \lesssim \mathrm{dist}(A,B)^{-\alpha}.$$

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Application to the long-range Kitaev chain

Kitaev chain :

$$H := -\,t\sum_{i=1}^{L}\left(a_i^\dagger\, a_{i+1} + \text{h.c.}\right) - \mu\sum_{i=1}^{L}\left(n_i - 1/2\right)$$
$$+\frac{\Delta}{2}\sum_{i=1}^{L}\left(a_i\, a_{i+1} + a_{i+1}^\dagger\, a_i^\dagger\right),$$

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Kitaev chain with long-range interactions [2, 3]:

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$$+ \sum_{i=1}^{L}\sum_{j=1}^{L-1} d_j^{-\alpha}\left(a_i\, a_{i+j} + a_{i+j}^\dagger\, a_i^\dagger\right),$$

Quadratic Hamiltonian, hence Wick's theorem implies

$$\mathrm{corr}_\beta(n_i, n_j) = \langle a_i^\dagger\, a_j\rangle_\beta\, \langle a_i\, a_j^\dagger\rangle_\beta - \langle a_i^\dagger\, a_j^\dagger\rangle_\beta\, \langle a_i\, a_j\rangle_\beta .$$

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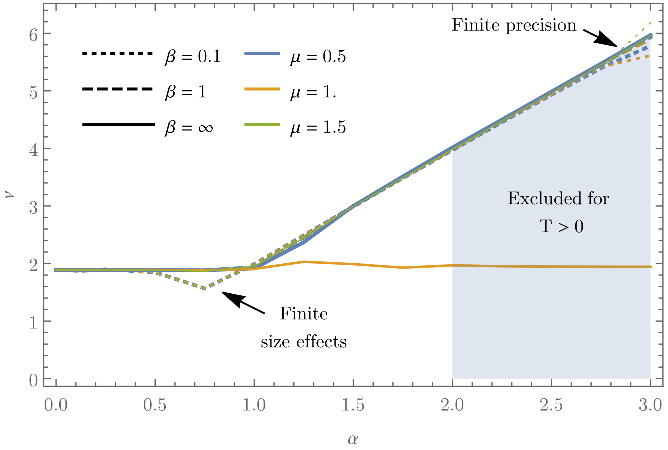
Application to the long-range Kitaev chain

Kitaev

Quad

Density-density correlations in a long-range Kitaev chain



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Methods

Combination of

- Integral representation of $\mathrm{corr}_\beta(A, B)$ [4] and

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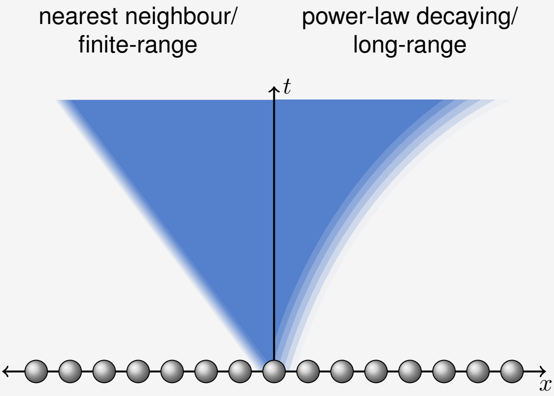
- Lieb-Robinson bounds [5]



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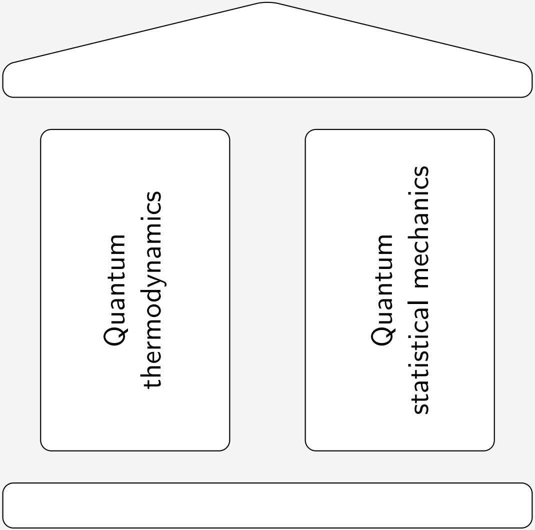
Motivation

Central question:

How difficult is it to bring a quantum many-body system permanently out of equilibrium?

Some context



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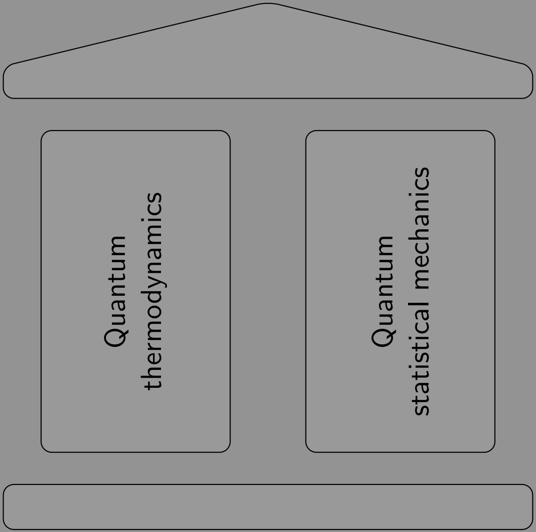

Equilibration

Equilibration

Theorem (Equilibration on average)

If H has non-degenerate energy gaps, then for every initial state $\rho = |\psi_0\rangle\langle\psi_0|$ there exists a state $\omega_H(\rho)$ such that

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[11] P. Reimann and M. Kastner, New J. Phys., 14.4 (2012), 43020
[12] C. Gogolin and J. Eisert, Reports Prog. Phys., 79.5 (2016), 56001

Equilibration

Theorem

If H has

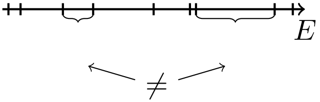
$\rho = |\psi_0\rangle\langle$

Non-degenerate energy gaps

H has non–degenerate energy gaps iff:

$$E_k - E_l = E_m - E_n$$

$$\implies k = l \wedge m = n \quad \vee \quad k = m \wedge l = n$$



Intuition: Sufficient for H to be fully interactive

$$H \neq H_1 \otimes \mathbb{1} + \mathbb{1} \otimes H_2$$

[7] M. Cramer, C.
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$$\overline{\big(\mathrm{Tr}(A\,\rho(t)) - \mathrm{Tr}(A\,\omega_H(\rho))\big)^2} \leq \frac{\|A\|_\infty^2}{d_H^{\mathrm{eff}}(\rho)}$$

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for all subsystems S
$$\overline{\mathcal{D}\left(\rho^S(t), \omega_H^S(\rho)\right)} \leq \frac{1}{2}\sqrt{\frac{d_S^2}{d_H^{\mathrm{eff}}(\rho)}}$$

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Effective dimension

$$d_H^{\mathrm{eff}}(\rho) := \frac{1}{\sum_k |\langle E_k|\psi_0\rangle|^4} = \frac{1}{\mathrm{Tr}(\omega_H(\rho)^2)}$$

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$$d_H^{\text{eff}}(\rho) \approx 2^{10^{23}}$$

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- It is huge for typical states from unitary invariant ensembles
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- Why the Rényi two entropy?

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$$S_\alpha(\omega) := \frac{1}{1-\alpha} \log(\mathrm{Tr}(\omega^\alpha))$$

- Alternatives? Yes! In terms of the second largest population [11].

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Motivation

Central questions:

- How difficult is it to avoid equilibration?
  Can we quantify this in a resource theoretic way?
- Which other equilibration bounds can we hope to prove?
  How arbitrary is the choice of the two entropy?

Preparing systems out of equilibrium

Given stationary states

$$\sigma^Q$$

$$\otimes$$

$$\sigma^R$$

Preparing systems out of equilibrium

Given
stationary
states

H_i^Q

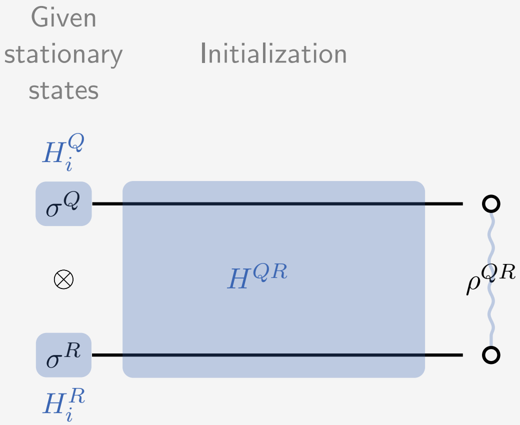
σ^Q

$\otimes$

σ^R

H_i^R

Preparing systems out of equilibrium



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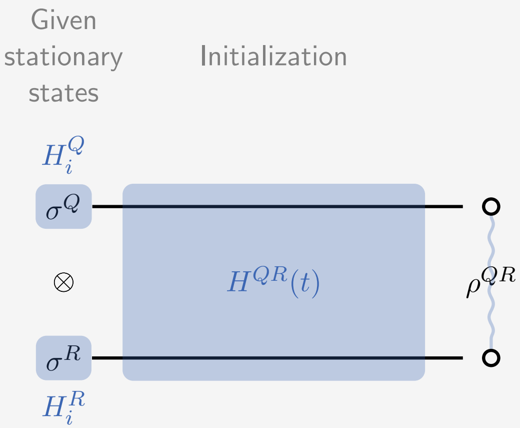

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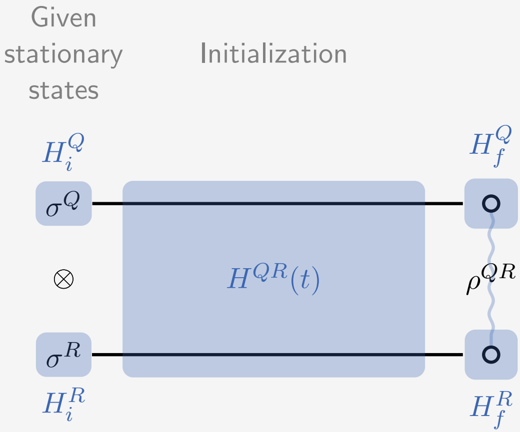

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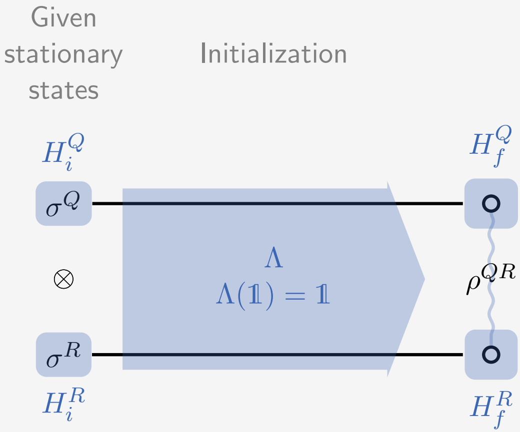

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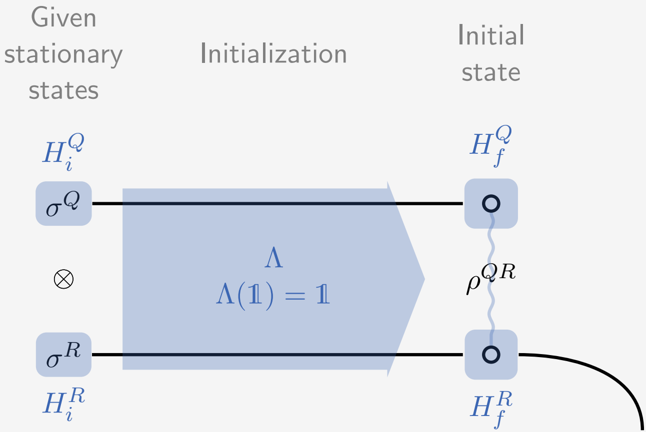

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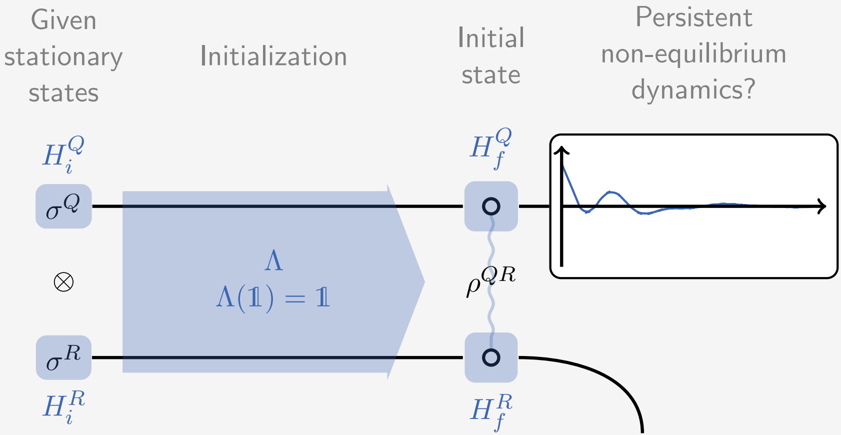

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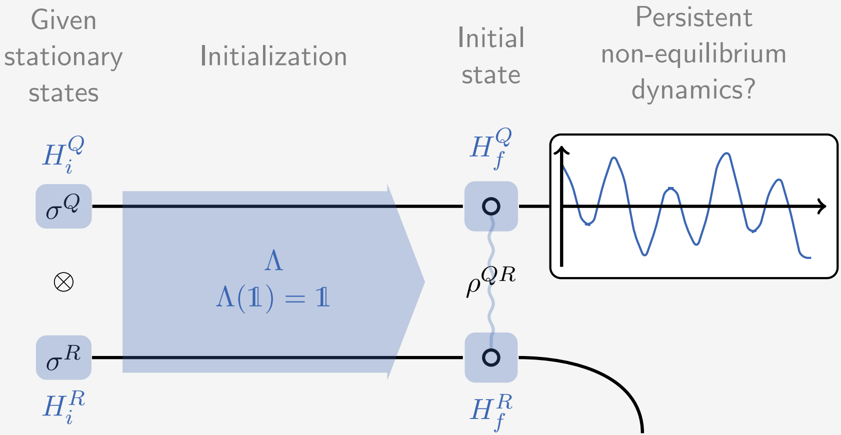

Resilience against equilibration

$$\mathcal{R}(\rho, H) := \log\left(\frac{d}{d_H^{\mathrm{eff}}(\rho)}\right)$$

Resilience against equilibration

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Properties:

- High resilience is necessary condition for avoiding equilibration.

Resilience against equilibration

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- Non-increasing under unital maps

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Results

The resilience as a resource

Theorem (No resilience for free)

Given $\sigma^Q \otimes \hat{\sigma}^R$ stationary and H_i^{QR} and H_f^{QR} non-interacting

$$\Delta \mathcal{R}^Q \leq \mathcal{R}(\sigma^R, H_i^R).$$

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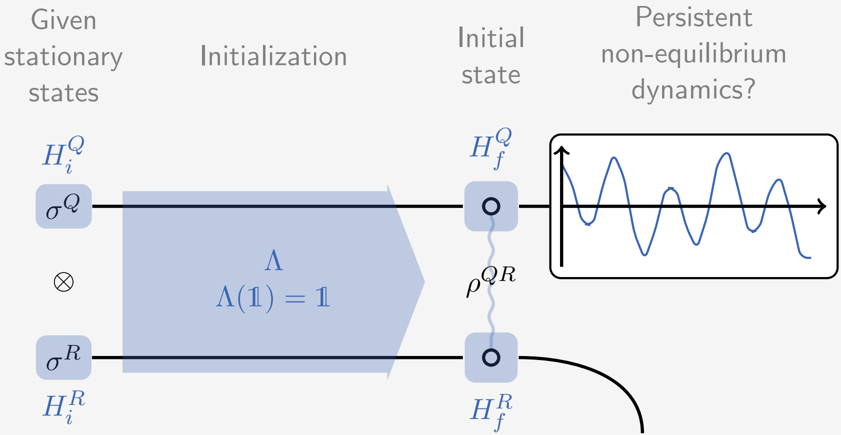
Doesn't mean we need to spend the resilience! But:

Theorem (Without correlations resilience is a resource)

If in addition $\rho^{QR} = \rho^Q \otimes \rho^R$

$$\Delta\mathcal{R}^Q \leq -\Delta\mathcal{R}^R.$$

Preparing systems out of equilibrium



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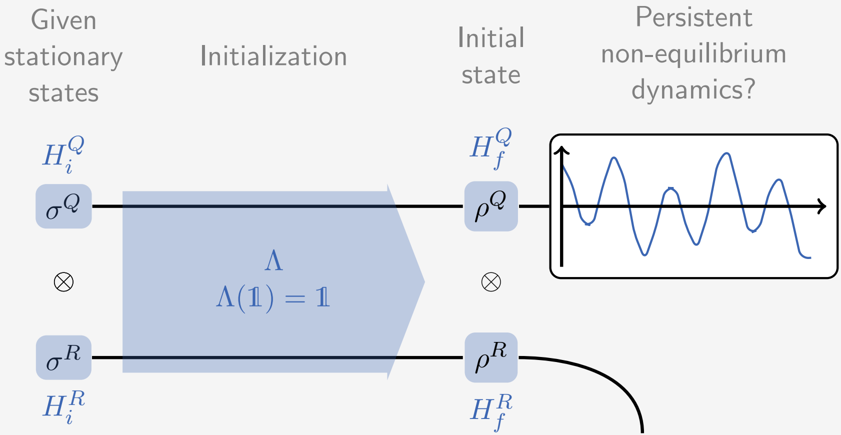

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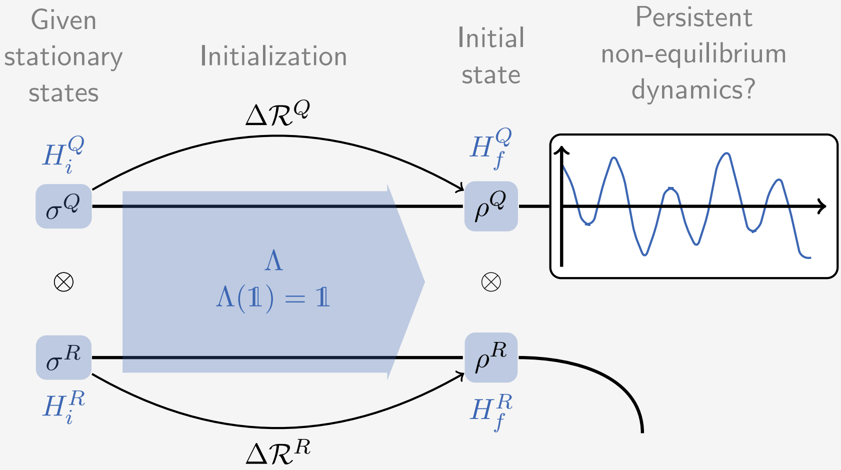

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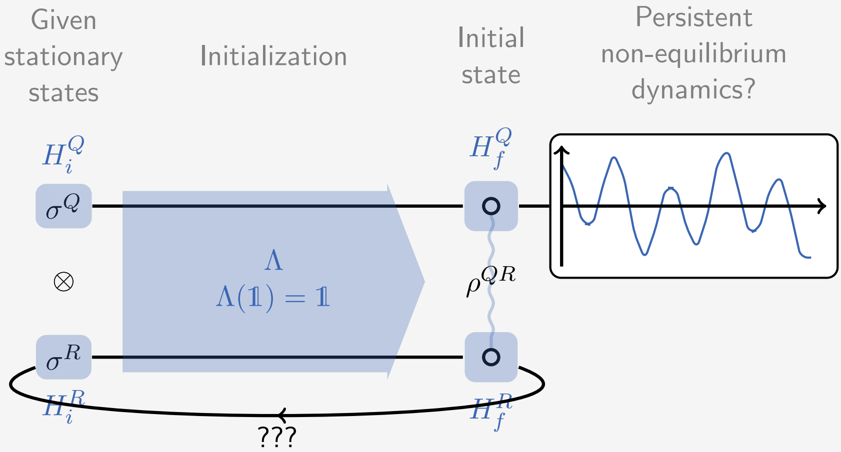

Catalytic processes

Consider a family of systems of increasing number of sub-systems n.

Theorem (No "second law of equilibration")

There are (natural) stationary states σ_n^Q and Hamiltonians such that for every $\epsilon > 0$ there exist states σ_n^R and a mixture of unitaries Λ such that:

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- Maybe physical restrictions on Λ can fix this?

Summary and outlook

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