Limits on non-local correlations from the structure of the local state space

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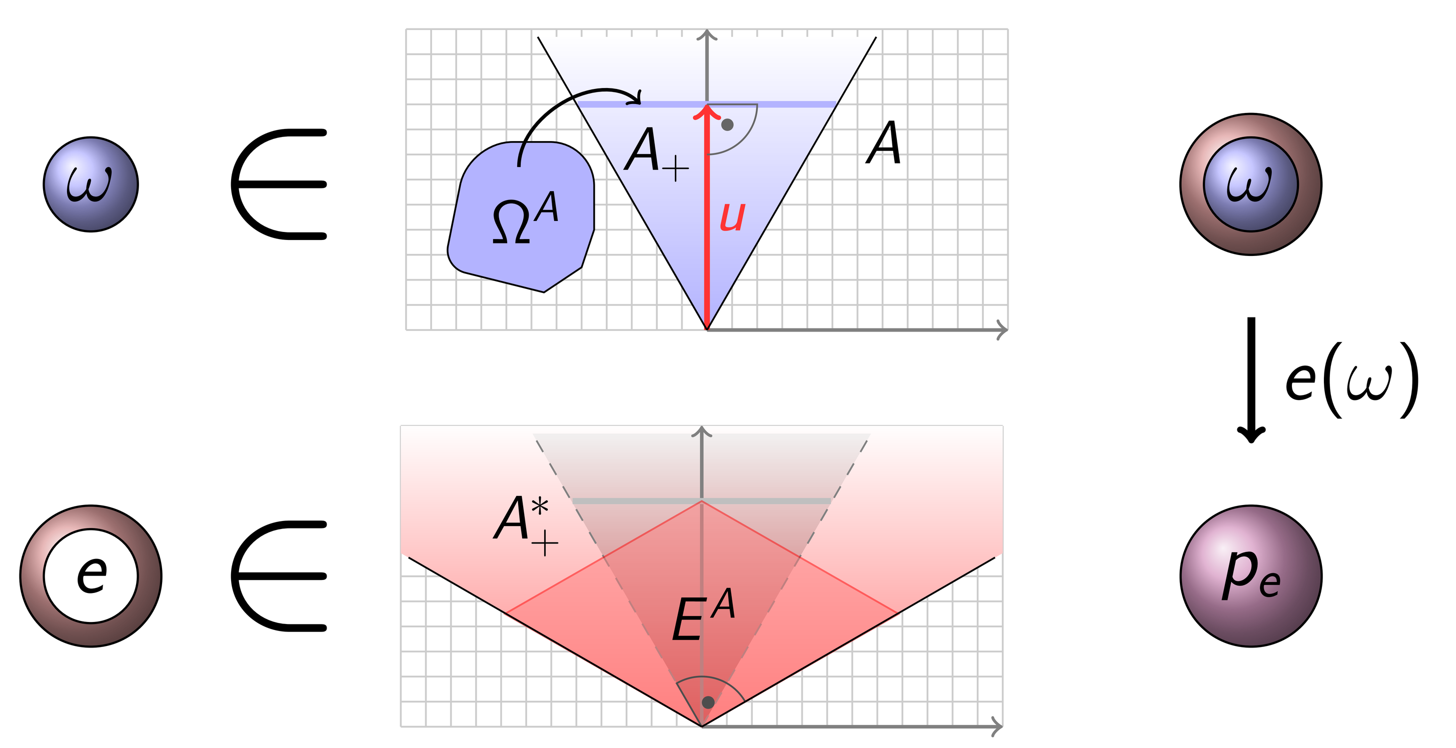
Generalized Probabilistic Theories [1]

Geometry of local state space Ω^A determines possible probability distributions



State space $\Omega^A \Rightarrow$ Possible measurement outcomes E^A

$$p_e = e(\omega) \in [0,1] \quad \forall e \in E^A, \omega \in \Omega^A$$

Largest joint state space Ω^{AB} of local systems Ω^A and Ω^B allowed by no-signaling: Maximal tensor product $\Omega^A \otimes_{\max} \Omega^B$

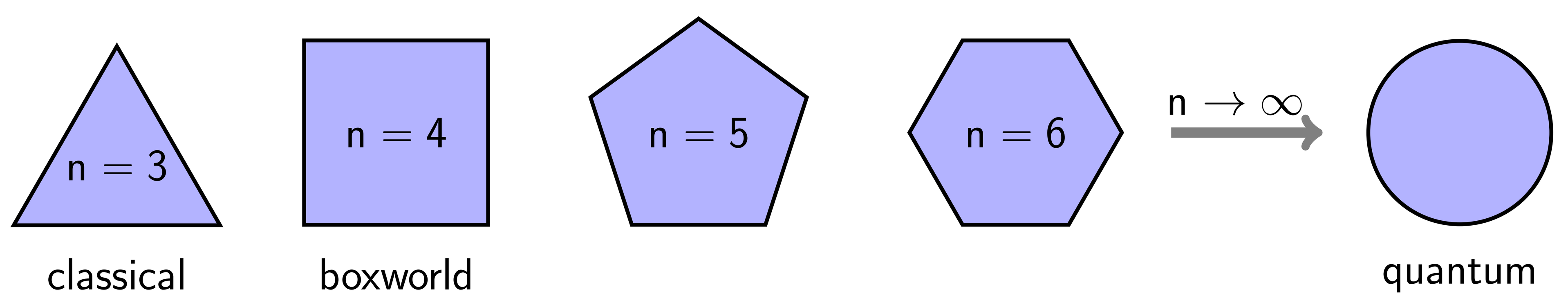
$$\Omega^A \otimes_{\max} \Omega^B = \{\omega^{AB} \in A \otimes B \,|\, \omega^{AB}(e^A \otimes e^B) \geq 0 \quad \forall e^A \in E^A, e^B \in E^B\}$$

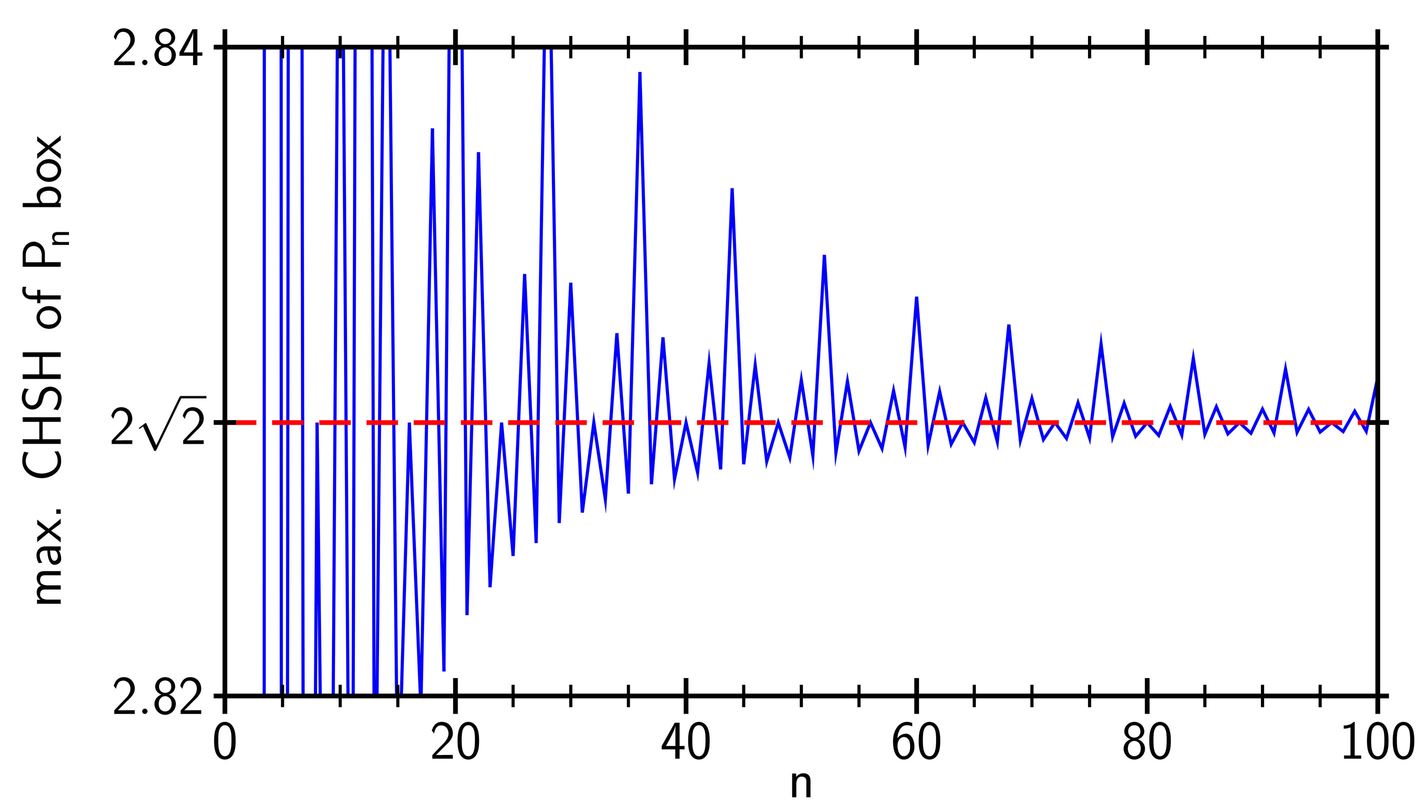
CHSH-coefficient S for two local binary measurements per site:

$$S = |E_{00} + E_{01} + E_{10} - E_{11}|$$

$$E_{AB} = \sum_{i=j} \omega^{AB}(e_i^A \otimes e_j^B) - \sum_{i \neq j} \omega^{AB}(e_i^A \otimes e_j^B)$$

Transition classical $\rightarrow$ quantum correlations by regular polygons with n vertices as local state spaces





Maximally entangled states $\Phi : e_i^B \rightarrow \omega_i^A$ are extremal in $\Omega^A \otimes_{\max} \Omega^B$ (for $n > 3$) [2]

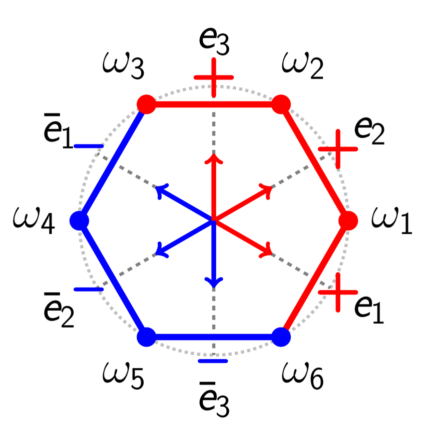
Polygon box or P_n box := Measurement statistics on maximally entangled state Φ of two polygon systems

Tsirelson's bound separates P_n boxes with even and odd n

Polygon boxes with even n

Braunstein-Caves Inequalities

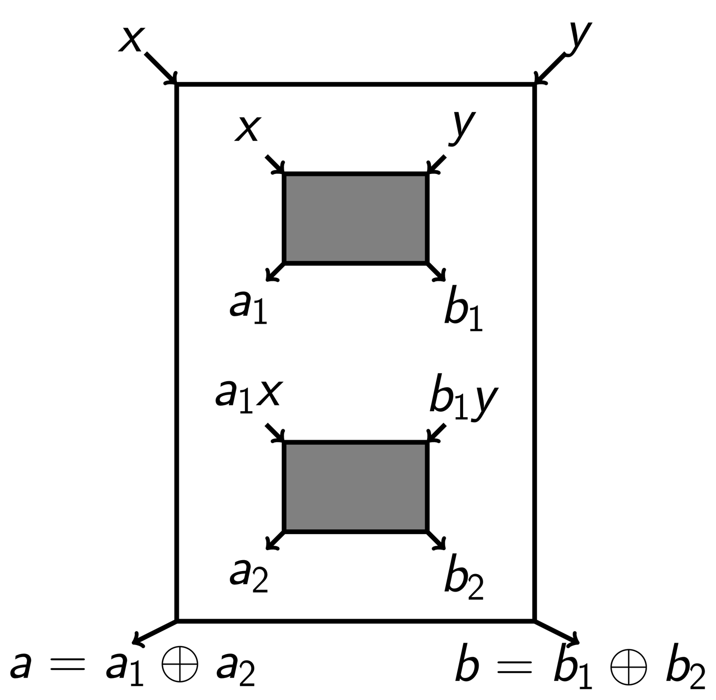
$$S_N = \left| \sum_{j=1}^{N-1} (E_{j,j} + E_{j,j+1}) + E_{N,N} - E_{N,1} \right| \leq 2N - 2$$

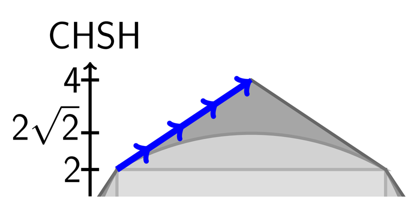


Maximal violation by polygon boxes with even n

$$E_{j,j} = 1 \qquad j = 1, \cdots, N$$
$$E_{j,j+1} = 1 \qquad j = 1, \cdots, N-1$$
$$E_{N,1} = -1$$

Nonlocality distillation possible using protocol of [4]





Polygon boxes with even n inherit powerful features of PR boxes

Drastically different to QM although almost no difference locally for high n

The set Q_1 [5, 6]

Best known approximation of the set of quantum correlations $Q \subset Q_1$

Connected to macroscopic locality

Closed under wirings, i.e. local processing cannot lead to correlations outside of the set

Respects Tsirelson's bound ($S \leq 2\sqrt{2}$)

Strongly self-dual subsystems

Regular polygons with odd n yield strongly self-dual state spaces

Strongly self-dual subsystems $\Rightarrow$ maximally entangled state Φ that defines an inner product

$$\Phi(e \otimes e) := \langle e, e \rangle$$

We showed that this implies that Φ yields correlations in Q_1

$\Rightarrow$ Polygon boxes with odd n show correlations in Q_1

$\Rightarrow$ Tsirelson's bound for the CHSH-violations of the maximally entangled states in QM can be regarded as a consequence of strong self-duality of single quantum systems

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