Generalized Probabilistic Theories

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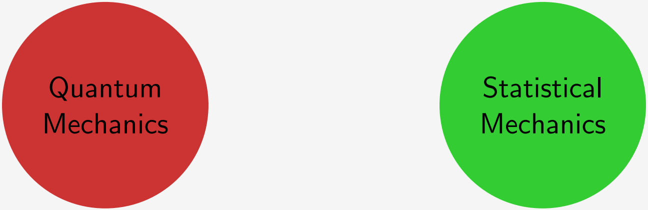
Motivation and background

Quantum Mechanics works, but it is not well understood!

Niels Bohr (1885-1962)

"Jeder, der von sich behauptet, er habe die Quantenmechanik verstanden, hat überhaupt nichts verstanden."

Better understanding by generalization



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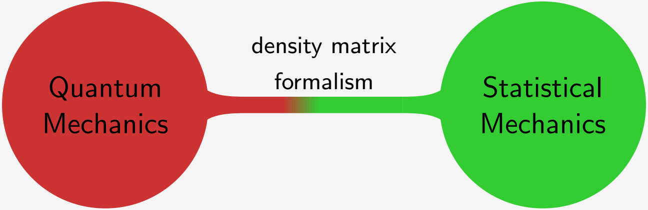

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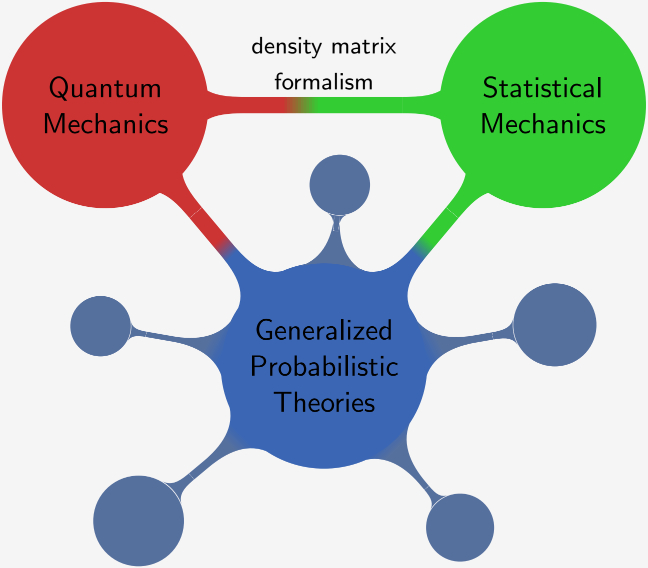

Taking a new viewpoint gives new insights

What can we learn from the GPT framework?

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- Why Quantum Mechanics?
- What are the alternatives?

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- Which properties of QM are genuine quantum?
  - Cloning is impossible in (almost) all non-classical GPTs
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  - Cloning is impossible in (almost) all non-classical GPTs
  - Broadcasting is impossible in (almost) all non-classical GPTs
  - Teleportation is possible in GPTs other that QM
- How to generalize concepts like entanglement or entropy?

Assumptions and fundamental concept

I. Isolated systems

An operational approach

Assumption

All one can learn about a given physical system is what one can learn by performing measurements on it.

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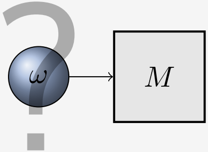

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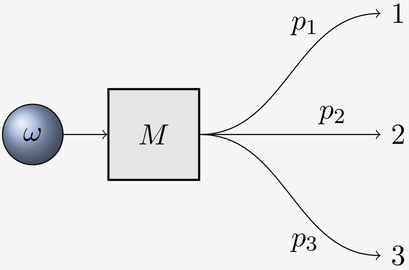

State

Definition

The state ω of a physical system is completely specified by giving the probabilities for the outcomes of all measurements that can be performed on it.
In turn, specifying the state ω, specifies all these probabilities.

$$\Longleftrightarrow \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ \dots \end{pmatrix}$$

"state space"

$$\Omega = \{\omega\}$$

Effects

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Every measurement outcome is associated with an effect e.
We write the probability to get this outcome when the system is in some state ω as $e(\omega) \in [0, 1]$.

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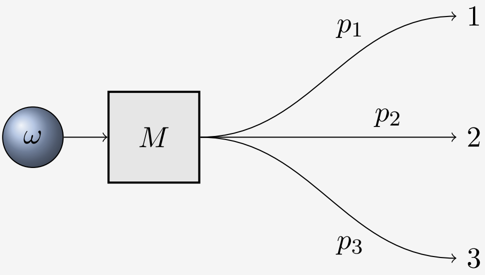

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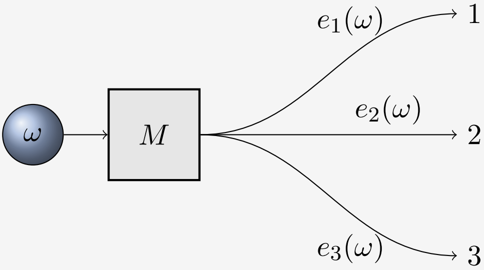

A certain measurement outcome

Unit measure

For every physical system there is a special effect, the so called unit measure u, defined by

$$u(\omega) = 1 \ \forall \ \omega \in \Omega$$

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"Is the system in one of its states?"

Mixed states

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Mixing two states ω_1 and ω_2 results in state that is a convex combination

$$\omega = p\,\omega_1 + (1-p)\,\omega_1,$$

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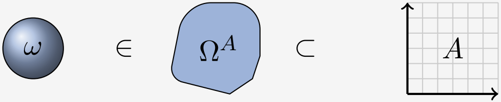
Definition

State that can not be written as a convex combination of states are called pure or extremal.

. . . and everything becomes linear . . .

1. States must be represented by elements of a linear space A.
2. The state space $\Omega^A \in A$ is convex and the extreme points of this set are the pure states.



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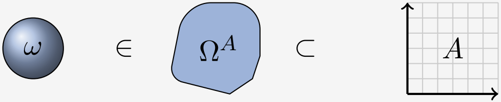
3. Effects are linear functionals on the state space $e : \Omega \rightarrow [0, 1]$.

$$e(\omega) = p\, e(\omega_1) + (1 - p)\, e(\omega_2)$$



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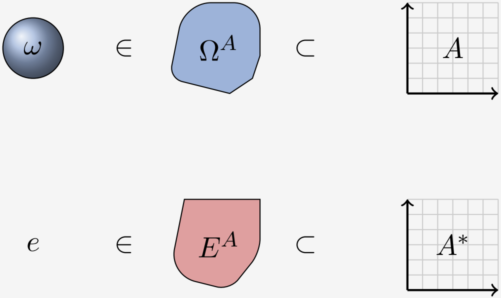
4. The effects form a convex set E^A in the dual space A^*.



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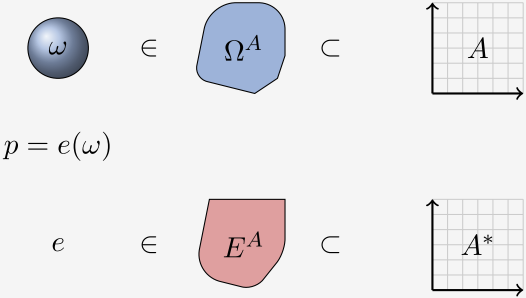

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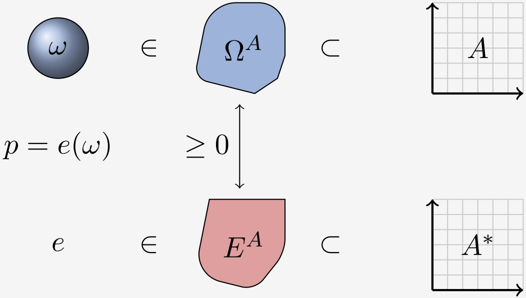


Assumptions and fundamental concept

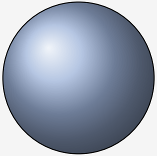
II. Joint systems

No-signalling

Assumption

Local operations on disjoint subsystems commute.

ω^{AB}



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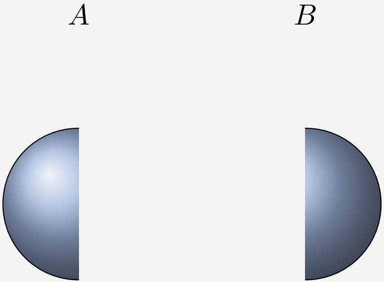

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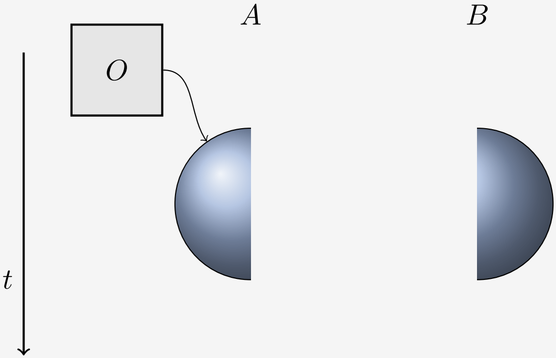

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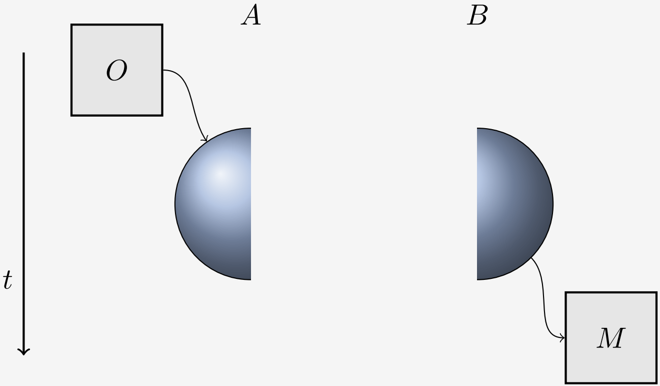

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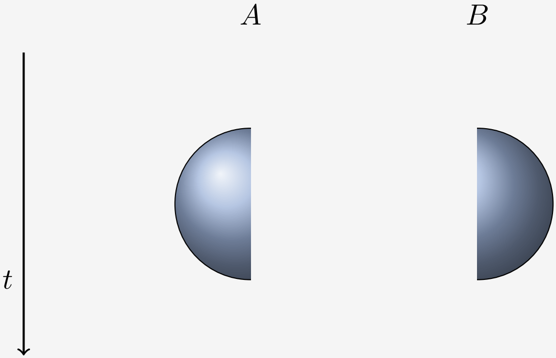

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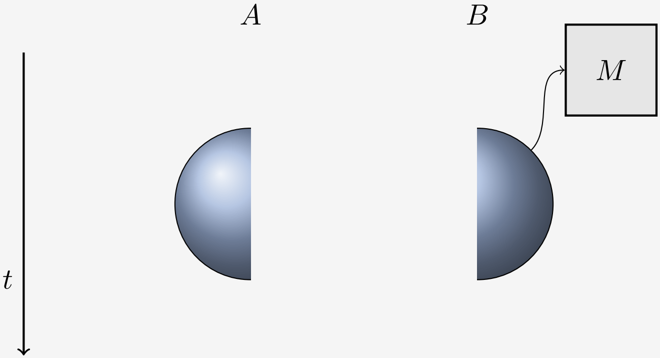

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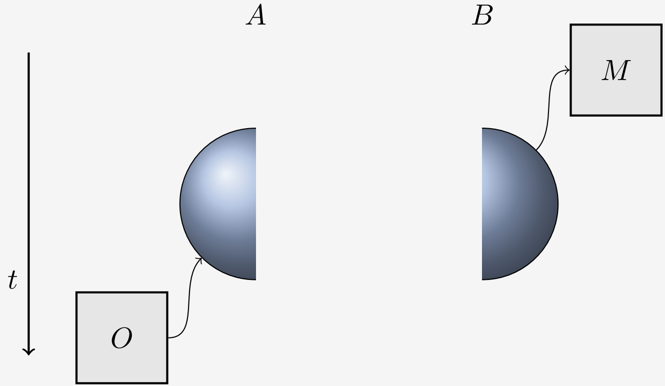

Fiducial measurements

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Giving the probabilities for all outcomes of a fiducial measurement is sufficient to uniquely specify a state.

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Fiducial measurements

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Giving the probabilities for all outcomes of a fiducial measurement is sufficient to uniquely specify a state.

$$\omega \Longleftarrow \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

Global state assumption

Assumption

Fiducial measurements on system A and B are sufficient to specify the state of the joint system AB.

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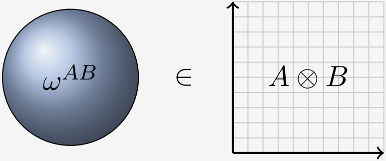
Global State Assumption
No-Signalling Principle
} $\Longrightarrow$



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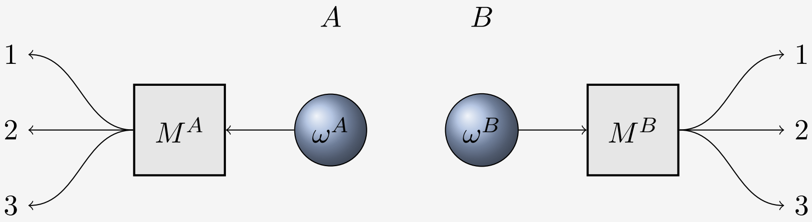

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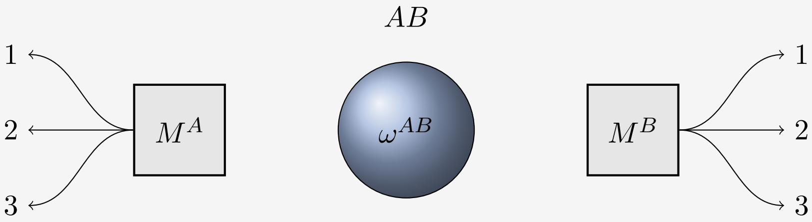

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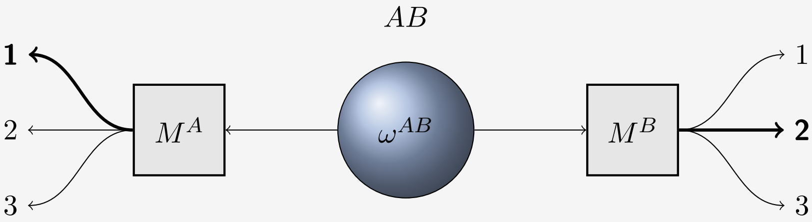
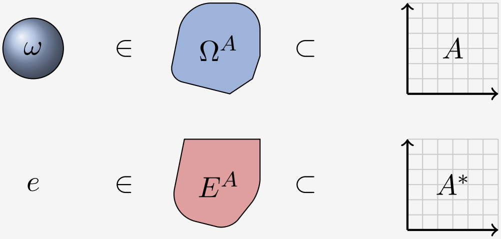


Summary and consequences

- Isolated systems:
  - Operational approach
  - Mixed states



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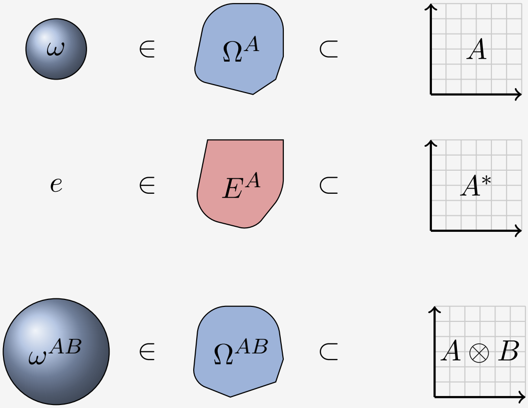


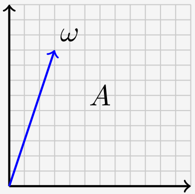
- Joint systems:
  - No-signalling
  - Global state

Mathematical representation

I. Isolated systems

Linear space A

- States ω are elements of a linear space A



Linear space A

- States ω are elements of a linear space A
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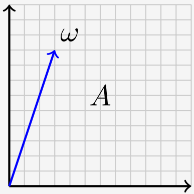

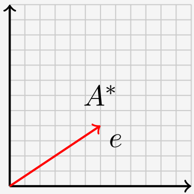

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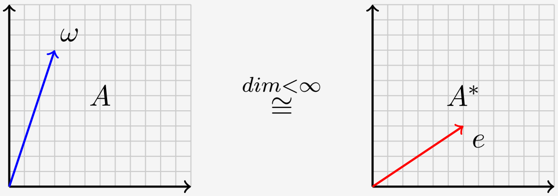
Special properties for A of finite dimension:

- A and A^* are self dual



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Special properties for A of finite dimension:

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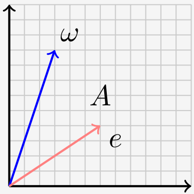
- $e(\omega)$ can be regarded as a scalar product



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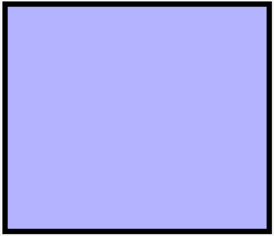
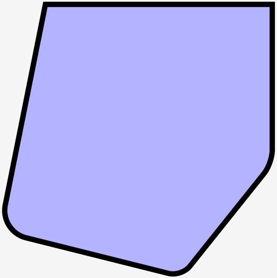
- There is also a geometric interpretation of convexity...
- All states ω on a strait line connecting $\omega_1, \omega_2 \in \Omega$ are part of Ω

Possible state spaces

- Ω can have an infinite number of extremal states





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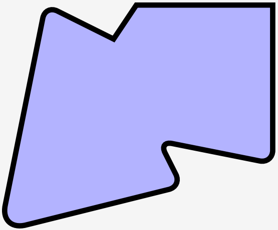
- Ω can have an infinite number of extremal states
- Bond of the state space must not bend to the inside



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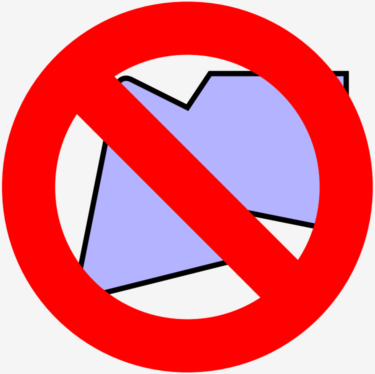

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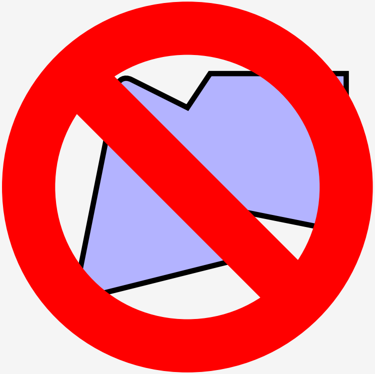

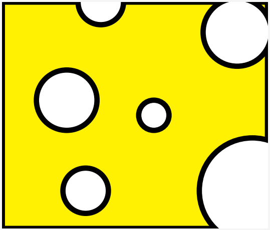

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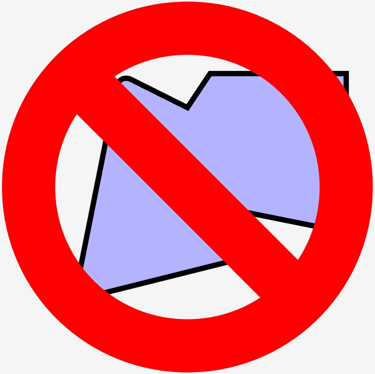

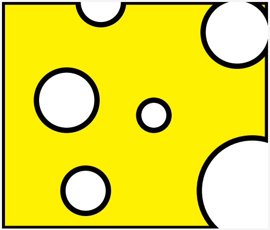

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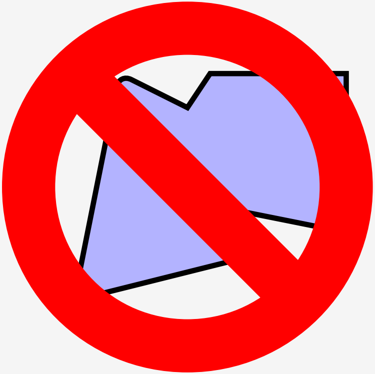
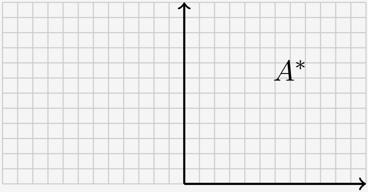


Choosing the unit measure u

- Next step to define a theory is to choose a unit measure:

$$u(\omega) = 1 \quad \forall\, \omega \in \Omega$$



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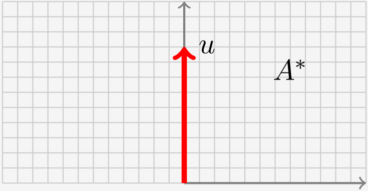

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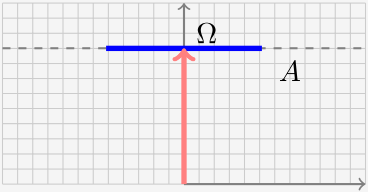
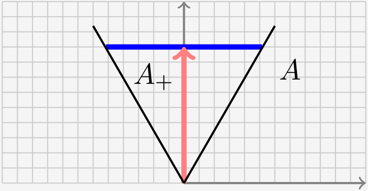
- Extra dimension for the unit measure
- State space in a hyperplane normal to u



Positive cone A_+

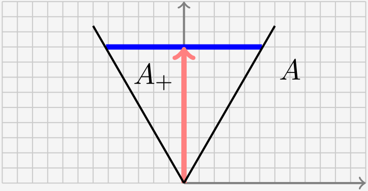
- Positive linear combinations of states $\omega \in \Omega$ construct a positive cone $A_+ \subset A$



Positive cone A_+

- Positive linear combinations of states $\omega \in \Omega$ construct a positive cone $A_+ \subset A$
- On the other hand:

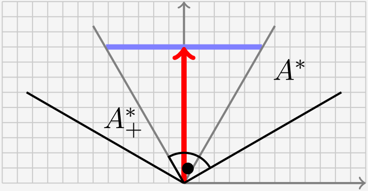
$$\Omega := \{\omega \in A_+ |\, u(\omega) = 1\}$$



Positive dual cone A_+^*

- It's dual cone $A_+^* \subset A^*$ is the set of e satisfying:

$$e(\omega) \geq 0 \quad \forall\, \omega \in \Omega$$



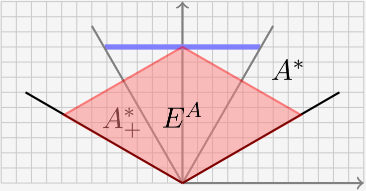
Positive dual cone A^*_+

- It's dual cone $A^*_+ \subset A^*$ is the set of e satisfying:

$$e(\omega) \geq 0 \quad \forall\, \omega \in \Omega$$

- The convex set $E^A \subset A^*_+$ of effects e is given by:

$$E_A := \left\{ e \in A^*_+ \,\middle|\, \sup_{\omega \in \Omega} e(\omega) \leq 1 \right\}$$



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$$M = \{e\} \qquad \sum_{e \in M} e = u$$

- Carrying out a measurement maps a state ω to a normalized probability distribution $\{p_e\}$ with $p_e = e(\omega)$
- The probability p_{any} to get any outcome is:

$$p_{any} = \sum p_e = \sum e(\omega) = u(\omega) = 1$$

Mathematical representation

II. Joint systems

Joint systems

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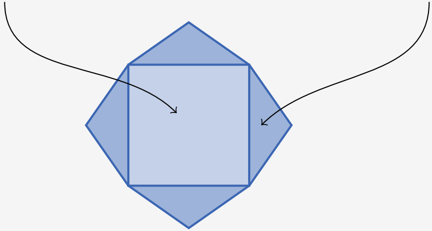

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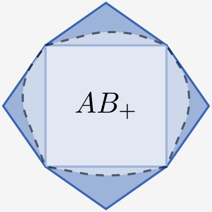
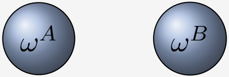


- A particular theory must specify the positive cone AB_+.

Tensor products



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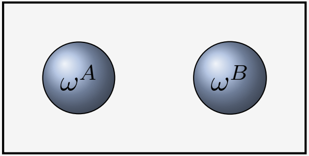

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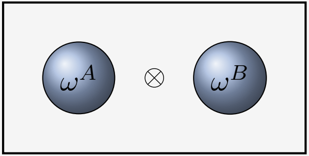

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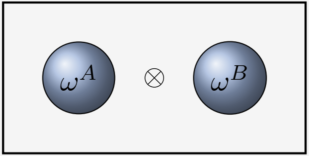

$$\omega_1^A \otimes \omega_1^B$$

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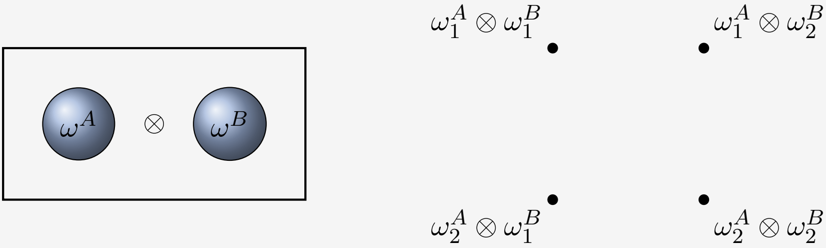

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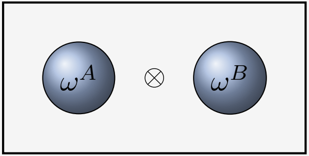

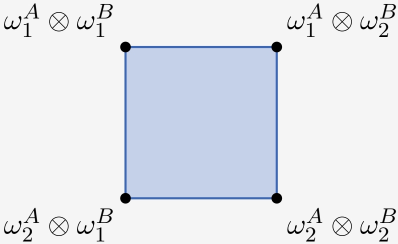

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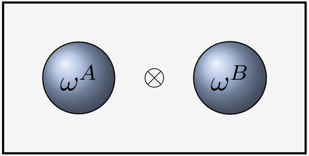

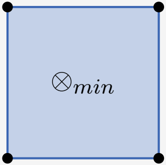

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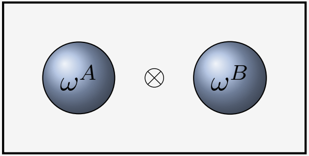

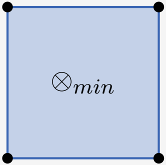

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- The same must hold for effects:

$$A_+ \otimes_{max} B_+ = \{\omega^{AB} \in A \otimes B \,|\, \omega^{AB}(e^A \otimes e^B) \geq 0\}$$

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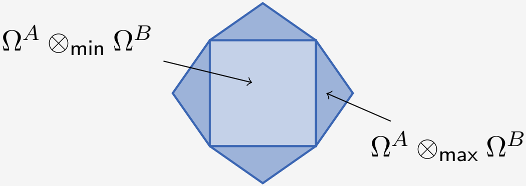

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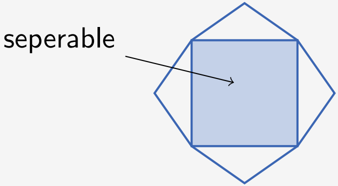

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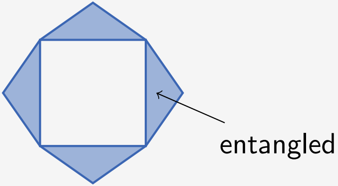

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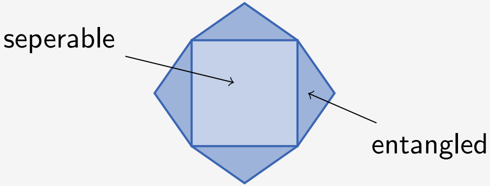


- A particular physical theory is given by a particular choice of:

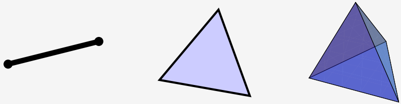
$$A_+, B_+, u^A, u^B \text{ and } AB_+$$

Examples

I. Classical probability theory

Classical probability theory

- State space Ω_{class} is a probability simplex



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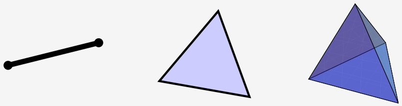

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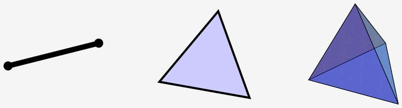

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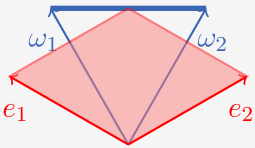

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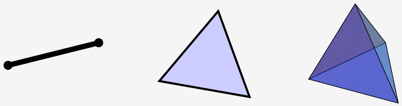

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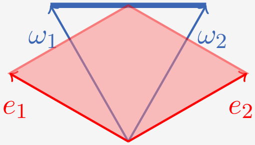

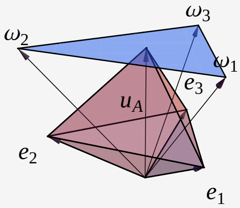

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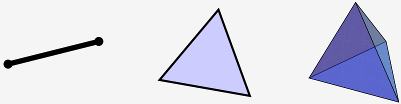

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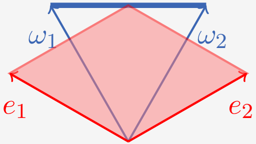

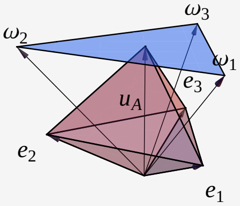

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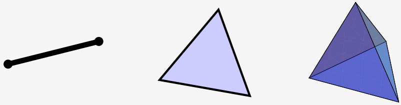

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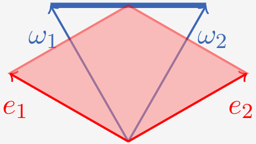

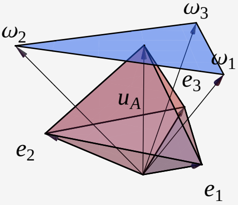

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$\Rightarrow$ Unique decomposition of mixed states

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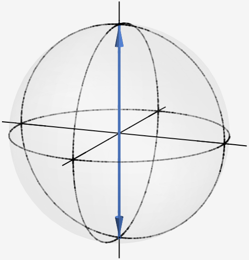
The Bloch sphere



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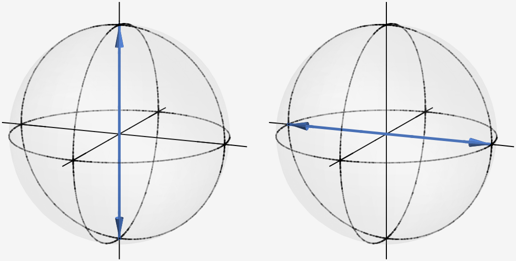

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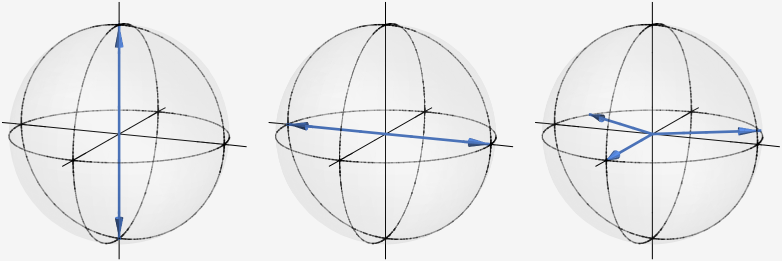

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- Entangled states ρ^{AB} not maximally correlated
- Mixed states in subsystems of pure entangled joint states

Examples

III. The Gbit

Building a GPT from scratch

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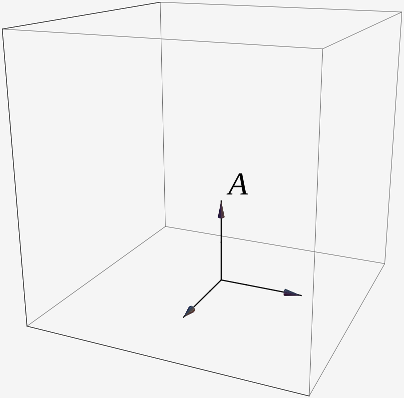
$$A = \left\{ \omega = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{pmatrix} \middle| \; \omega_{11} + \omega_{12} = \omega_{21} + \omega_{22} = c \right\}$$



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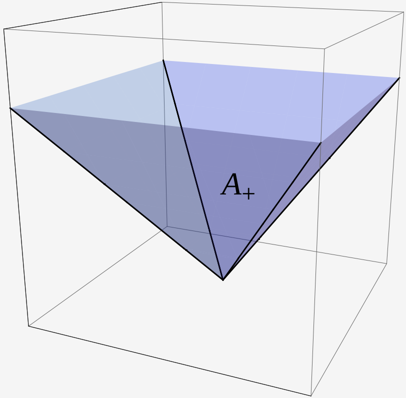

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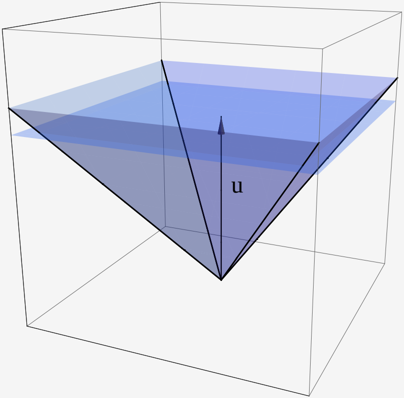
$$u = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$$u(\omega) = \mathrm{tr}(u^\dagger \, \omega) \overset{!}{=} 1$$



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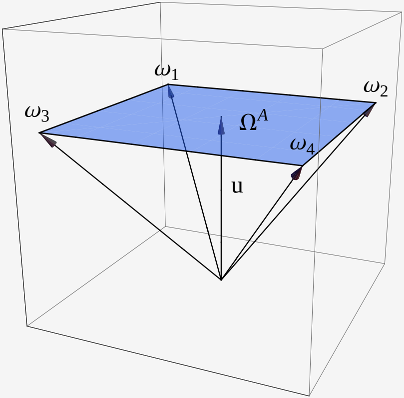
$$\Omega^A = \left\{ \omega = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{pmatrix} \middle| \; \omega_{11} + \omega_{12} = \omega_{21} + \omega_{22} = 1, \; \omega_{ij} \geq 0 \right\}$$

$$\omega_1 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \quad \omega_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\omega_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \omega_4 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$



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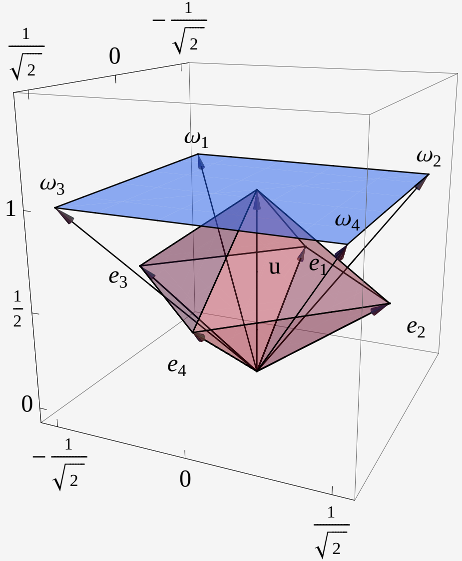
$$e_1 = \frac{1}{4}\begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix} \; e_2 = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} \; e_3 = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \; e_4 = \frac{1}{4}\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$$

States and effects



Thank you for your attention!

Literatur

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A generalized no-broadcasting theorem, 2007.

[2] Howard Barnum, Jonathan Barrett, Matthew Leifer, and Alexander Wilce.
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$\longrightarrow$ Beamer slides: http://www.cgogolin.de