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Einselection without pointer states

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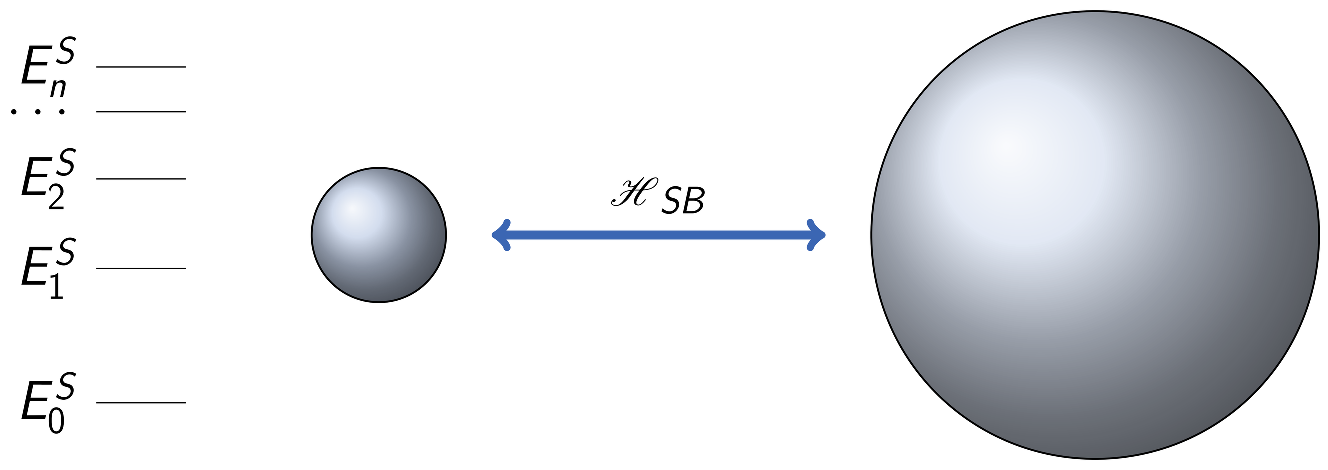
Setup: A quantum system interacting with a much larger environment:

System, $\mathcal{H}_S$ Bath, $\mathcal{H}_B$



$\rho_t^S = \mathrm{Tr}_B[\psi_t]$ $\rho_t^B = \mathrm{Tr}_S[\psi_t]$

$d_S = \dim(\mathcal{H}_S)$ $d_B = \dim(\mathcal{H}_B)$

Question:

What can we say about the time development of the quantum system under minimal assumptions?

No added randomness: Bipartite quantum system with unitary time evolution and a pure global state:

$$|\psi_t\rangle \in \mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_B$$

Hamiltonian: No restrictions on the interaction:

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_S \otimes \mathbb{1} + \mathbb{1} \otimes \mathscr{H}_B + \mathscr{H}_{SB}$$

$$\mathscr{H}_0 \propto \mathbb{1} \qquad \mathrm{Tr}[\mathscr{H}_S] = \mathrm{Tr}[\mathscr{H}_B] = \mathrm{Tr}[\mathscr{H}_{SB}] = 0$$

One very weak assumption: "non-degenerate energy gaps"

$$E_k - E_l = E_m - E_n$$
$$\Longrightarrow k = l \wedge m = n \ \vee \ k = m \wedge l = n$$

Important quantity: Effective dimension of the time averaged state $\omega = \langle \rho_t \rangle_t$:

$$d^{\mathrm{eff}}(\omega) = \frac{1}{\mathrm{Tr}[\omega^2]}$$

What is known:

Equilibration

Theorem 2 in [2]: For a random pure state $|\psi_0\rangle \in \mathcal{H}$, the probability that $d^{\mathrm{eff}}(\omega)$ is smaller than $d/4$, $d = \dim(\mathcal{H})$ is exponentially small:

$$\Pr\left\{ d^{\mathrm{eff}}(\omega) < \frac{d}{4} \right\} \leq 2\,\mathrm{e}^{-C\sqrt{d}}$$

Theorem 1 in [2]: The reduced state ρ_t^S is close to its time average ω^S if $d^{\mathrm{eff}}(\omega) \gg d_S^2$:

$$\langle \mathcal{D}(\rho_t^S, \omega^S) \rangle_t \leq \frac{1}{2}\sqrt{\frac{d_S^2}{d^{\mathrm{eff}}(\omega)}}$$

Speed of fluctuations around equilibrium

Speed of the reduced state:

$$v_S(t) = \lim_{\delta t \to 0} \frac{\mathcal{D}(\rho_t^S, \rho_{t+\delta t}^S)}{\delta t} = \frac{1}{2}\left\| \frac{d\rho_t^S}{dt} \right\|_1 \qquad \frac{d\rho_t^S}{dt} = \mathrm{i}\,\mathrm{Tr}_B[\rho_t, \mathscr{H}]$$

Theorem from [3]: The reduced state is slow if $d^{\mathrm{eff}}(\omega) \gg d_S^3$:

$$\langle v_S(t) \rangle_t \leq \| \mathscr{H}_S \otimes \mathbb{1} + \mathscr{H}_{SB} \|_\infty \sqrt{\frac{d_S^3}{d^{\mathrm{eff}}(\omega)}}$$

Decoherence à la Zurek

Needs a special Hamiltonian with pointer sates $|p\rangle$ [4]:

$$\mathscr{H} = \sum_p |p\rangle\langle p| \otimes \mathscr{H}^{(p)}$$
$$U_t = \sum_p |p\rangle\langle p| \otimes U_t^{(p)}$$

Off-diagonal elements in the pointer basis are suppressed (einselection):

$$\langle p|\rho_t^S|p'\rangle = \langle p|\rho_0^S|p'\rangle\, \langle \psi_0^B| U_t^{(p')\dagger}\, U_t^{(p)} |\psi_0^B\rangle$$

New results:

Einselection without pointer states

Consider the experimentally relevant situation of weak coupling to the environment.

Examples:

- Electronic excitations of gases at moderate temperature
- Radioactive decay
- Quantum information processing

A generic weak interaction causes decoherence in the eigenbasis of the local Hamiltonian:

Theorem 4 in [1]: All reduced states ρ^S which are slow in the sense that

$$\left\| \frac{d\rho^S}{dt} \right\|_1 \leq \epsilon$$

satisfy

$$\max_{k \neq l} 2\,|E_k^S - E_l^S|\,|\rho_{kl}^S| \leq 2\,\| \mathscr{H}_{SB} \|_\infty + \epsilon$$

where $\rho_{kl}^S = \langle E_k^S|\rho^S|E_l^S\rangle$ and E_k^S and $|E_k^S\rangle$ are eigenvalues/eigenstates of $\mathscr{H}_S$.

Consequences:

- Assume weak interaction $\| \mathscr{H}_{SB} \|_\infty \ll |E_k^S - E_l^S|$

$\Longrightarrow$ Coherent superposition of $|E_k^S\rangle$ and $|E_l^S\rangle$ are destroyed.

$\Longrightarrow$ ρ_t^S is almost diagonal in the $\mathscr{H}_S$ eigenbasis most of the time.

Advantages:

- No special assumptions on the Hamiltonian or bath (as opposed to Zurek)
- Very general statements with broad applicability

Open problems:

- Time for decoherence
- Time for equilibration

References:

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[3] N. Linden, S. Popescu, A. J. Short, and A. Winter, "On the speed of fluctuations around thermodynamic equilibrium," arXiv:0907.1267v1.

[4] W. H. Zurek, "Environment-induced superselection rules," *Phys. Rev. D* (1982) no. 26, 1862–1880.