Einselection without pointer states

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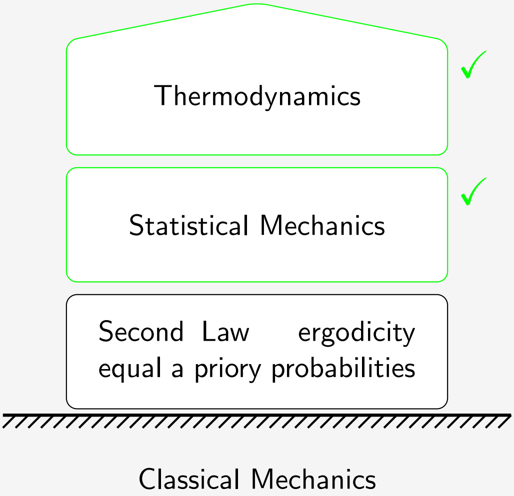
Decoherence under weak interaction

Christian Gogolin

Universität Würzburg

2009-12-16

New foundation for Statistical Mechanics



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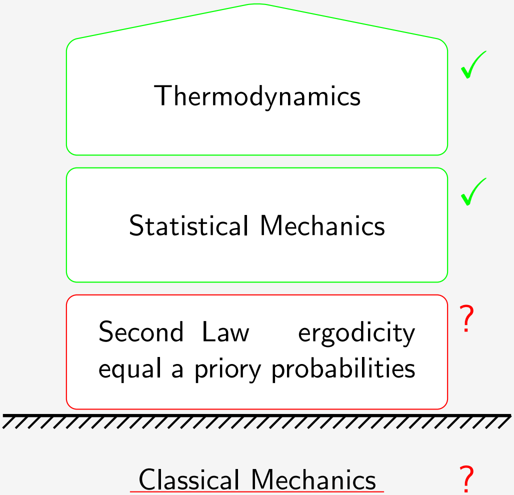

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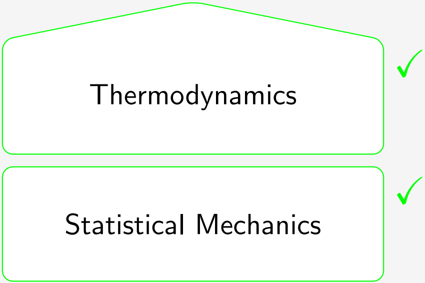

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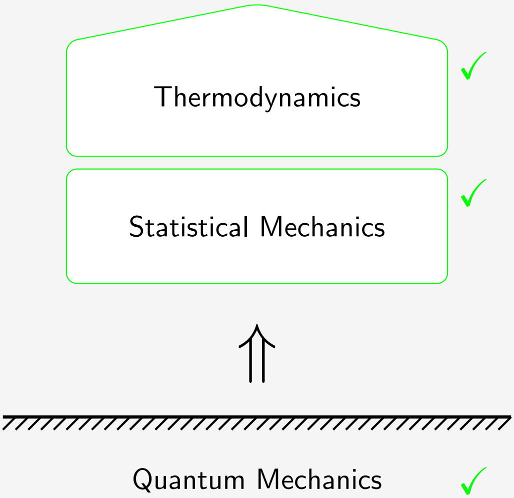

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Why do electrons hop between energy eigenstates?

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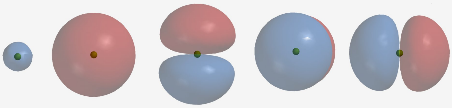
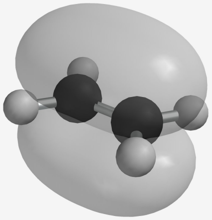
quantum mechanical orbitals





coherent superpositions

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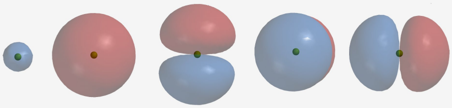
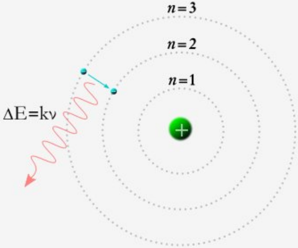
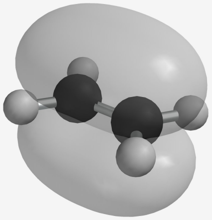


discrete energy levels





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hopping

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Technical introduction

Quantum Mechanics on one slide

- Pure Quantum Mechanics

$$|\psi\rangle \in \mathcal{H} \qquad A = A^\dagger$$

$$\langle\psi|\psi\rangle = 1 \qquad \langle A\rangle_\psi = \langle\psi|A|\psi\rangle$$

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mixtures: $\rho = p\,\psi_1 + (1-p)\,\psi_2$

Technical introduction

- Trace distance

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- Time average

$$\omega = \langle \rho_t \rangle_t = \lim_{T\to\infty} \frac{1}{T}\int_0^T \rho_t\, dt$$

Choosing random states

- Mathematical construction

Haar measure on $SU(n)$ $\longrightarrow$ "uniform" distribution

$$\mu(V) = \mu(U\,V) \qquad U\,|0\rangle = |\psi\rangle \qquad \Pr\{|\psi\rangle\} = \Pr\{U\,|\psi\rangle\}$$

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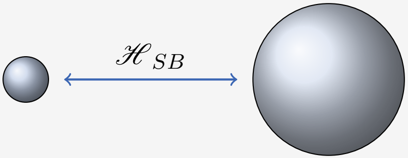
- Explicit construction
  1. expand in basis: $|\psi\rangle = \sum_i c_i\,|i\rangle \qquad \langle i|j\rangle = \delta_{ij}$
  2. choose c_i from a normal distribution
  3. normalize $1 = \sum_i |c_i|^2 \qquad \langle\psi|\psi\rangle = 1$

Setup

Setup

System, $\mathcal{H}_S, \mathscr{H}_S$ Bath, $\mathcal{H}_B, \mathscr{H}_B$



$\rho_t^S = \mathrm{Tr}_B[\psi_t]$ $\rho_t^B = \mathrm{Tr}_S[\psi_t]$

$$\mathrm{Tr}[(A_S \otimes \mathbb{1}_B)\psi_t] = \mathrm{Tr}[A_S\, \rho_t^S]$$

reduced state $\rightarrow$ locally observable

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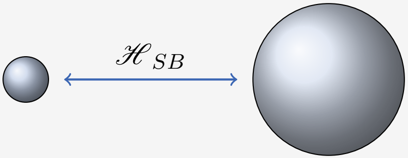

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$$\frac{d\psi_t}{dt} = \mathrm{i}\,[\psi_t, \mathscr{H}]$$

A very weak assumption on the Hamiltonian

$$\mathscr{H} = \mathscr{H}_S \otimes \mathbb{1} + \mathbb{1} \otimes \mathscr{H}_B + \mathscr{H}_{SB}$$

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Assumption

A Hamiltonian has non–degenerate energy gaps iff:

$$E_k - E_l = E_m - E_n$$

$$\Longrightarrow k = l \wedge m = n \text{ or } k = m \wedge l = n$$

Subsystem equilibration and fluctuations around equilibrium

Measure concentration in Hilbert space

Theorem 1

For random $\psi_0 \in \mathcal{P}_1(\mathcal{H})$ *with* $d = \dim(\mathcal{H})$

$$\Pr\left\{d^{\text{eff}}(\omega) < \frac{d}{4}\right\} \leq 2\,\mathrm{e}^{-c\sqrt{d}}$$

$d^{\text{eff}}(\omega) \sim$ # energy eigenstates in $|\psi_0\rangle$

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$\Longrightarrow$ If d is large then $d^{\text{eff}}(\omega)$ is large.

[6]

Equilibration

Theorem 2

For every $\psi_0 \in \mathcal{P}_1(\mathcal{H})$

$$\langle \mathcal{D}(\rho_t^S, \omega^S) \rangle_t \leq \frac{1}{2} \sqrt{\frac{d_S^2}{d^{\text{eff}}(\omega)}}$$

where

$$\rho_t^S = \operatorname{Tr}_B \psi_t \qquad \omega^S = \langle \rho_t^S \rangle_t \qquad \omega = \langle \psi_t \rangle_t$$

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$\Longrightarrow$ If $d^{\text{eff}}(\omega) \gg d_S^2$ then ρ_t^S equilibrates.

[6]

Speed of the fluctuations around equilibrium

$$v_S(t) = \lim_{\delta t \to 0} \frac{\mathcal{D}(\rho_t^S, \rho_{t+\delta t}^S)}{\delta t} = \frac{1}{2} \left\| \frac{d\rho_t^S}{dt} \right\|_1$$

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Theorem 3

For every $\psi_0 \in \mathcal{P}_1(\mathcal{H})$

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$\implies$ If $d^{\text{eff}}(\omega) \gg d_S^3$ then ρ_t^S is slow.

[5]

Summary

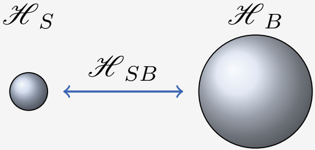
Typical states of large quantum systems

- have a high average effective dimension,
- their subsystems equilibrate
- and fluctuate slowly around the equilibrium state.

Decoherence under weak interaction

Approach 1: Effective dynamics

standard QM:



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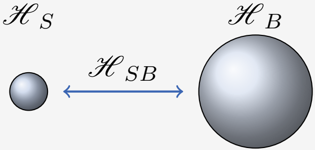

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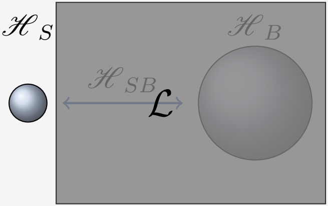

$$\frac{d\rho_t^S}{dt} = \mathrm{i}\,[\rho_t^S, \mathscr{H}_S] + \mathrm{i}\ \mathcal{L}(\rho_t^S)$$

Approach 2: Decoherence à la Zurek

- Special Hamiltonian with pointer sates $|p\rangle$:

$$\mathscr{H} = \sum_p |p\rangle\langle p| \otimes \mathscr{H}^{(p)}$$

- Initial product state $\psi_0 = \rho_0^S \otimes \psi_0^B$

[7]

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Einselection

Off-diagonal elements in the pointer basis are suppressed:

$$\langle p|\rho_t^S|p'\rangle = \langle p|\rho_0^S|p'\rangle \underbrace{\langle \psi_0^B|U_t^{(p')\dagger} U_t^{(p)}|\psi_0^B\rangle}_{\leq 1}$$

[7]

Comparison

Pros and cons

	unitary evolution	general mechanism
effective dynamics	X	✓
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Can we find a more general mechanism based on standard Quantum Mechanics?

Yes we can!

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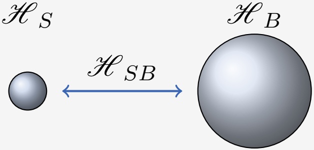
$$\| \mathscr{H}_{SB} \|_\infty \ll \| \mathscr{H}_S \|_\infty,$$



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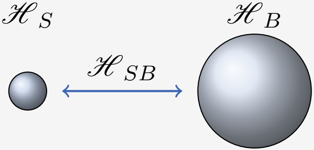
when is the subsystem slow ?

$$v_S(t)\frac{1}{2} = \left\| \frac{d\rho_t^S}{dt} \right\|_1$$



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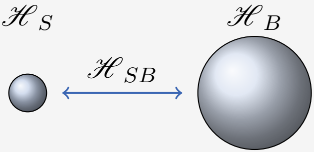

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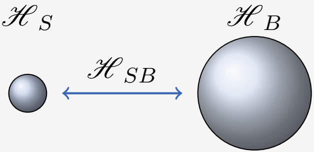

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$$= \mathrm{i}\ \mathrm{Tr}_B[\psi_t, \mathscr{H}_0 + \mathscr{H}_S \otimes \mathbb{1} + \mathbb{1} \otimes \mathscr{H}_B + \mathscr{H}_{SB}]$$

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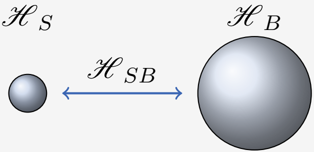

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Tow competing forces

$$\frac{d\rho_t^S}{dt} = \mathrm{i}\ \mathrm{Tr}_B[\psi_t, \mathscr{H}_S \otimes \mathbb{1} + \mathscr{H}_{SB}] = \mathrm{i}\,[\rho_t^S, \mathscr{H}_S] + \mathrm{i}\ \mathrm{Tr}_B[\psi_t, \mathscr{H}_{SB}]$$

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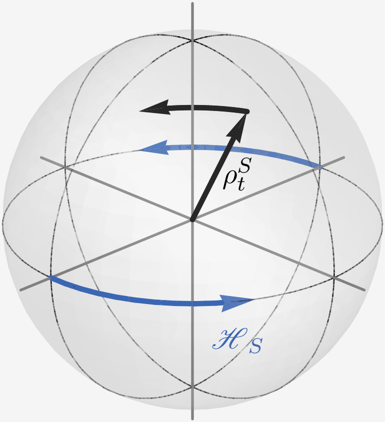

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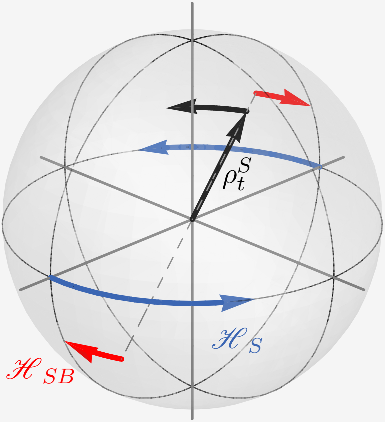

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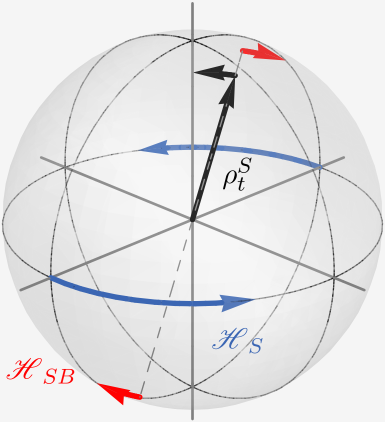

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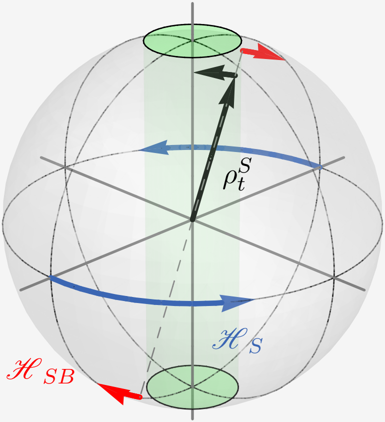

Decoherence through weak interaction

Theorem 4

All reduced states ρ_t^S satisfy

$$\max_{k\neq l} 2\,|E_k^S - E_l^S|\,|\rho_{kl}^S| \leq 2\,\|\,\mathscr{H}_{SB}\,\|_\infty + \left\|\frac{d\rho_t^S}{dt}\right\|_1$$

where

$$\rho_{kl}^S = \langle E_k^S|\rho_t^S|E_l^S\rangle$$

[1]

Decoherence through weak interaction

Theorem 4

All reduced states ρ_t^S *satisfy*

$$\max_{k \neq l} 2\,|E_k^S - E_l^S|\,|\rho_{kl}^S| \leq 2\,\|\mathscr{H}_{SB}\|_\infty + \left\|\frac{d\rho_t^S}{dt}\right\|_1$$

where

$$\rho_{kl}^S = \langle E_k^S|\rho_t^S|E_l^S\rangle$$

$\Longrightarrow$ If ρ_t^S is slow its off-diagonal elements are small.

[1]

Conclusions

Pros and cons

	unitary evolution	general mechanism
effective dynamics	X	✓
einselection	✓	X
our mechanism	✓	✓

Applications

- electronic excitations of gases at moderate temperature
- radioactive decay
- environment assisted entanglement creation
- ...

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Thank you for your attention!

$\longrightarrow$ beamer slides: http://www.cgogolin.de