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Efficient simulation of dissipative quantum dynamics on a quantum computer

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We show that the time evolution of an open quantum system, described by a possibly time dependent Liouvillian, can be simulated by a unitary quantum circuit of a size scaling polynomially in the simulation time and the size of the system. An immediate consequence is that dissipative quantum computing is no more powerful than the unitary circuit model. Our result can be seen as a dissipative Church-Turing theorem, since it implies that under natural assumptions, such as weak coupling to an environment, the dynamics of an open quantum system can be simulated efficiently on a quantum computer. Formally, we introduce a Trotter decomposition for Liouvillian dynamics and give explicit error bounds. This constitutes a practical tool for numerical simulations, e.g., using matrix-product operators. We also demonstrate that most quantum states cannot be prepared efficiently.

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Introduction

Strong quantum Church-Turing thesis

Every quantum mechanical computational process can be simulated efficiently in the unitary circuit model of quantum computation.

We show that this famous thesis rigorously holds in a quite general setting.

Assumptions

- The system is Markovian, by this we mean the system state ρ evolves according to
$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \mathcal{L}_t(\rho(t)) \ .$$
- N subsystems of Hilbert space dimension d
- Liouvillian is k-local, i.e.
$$\mathcal{L} = \sum_{\Lambda\subset[N]} \mathcal{L}_\Lambda \quad \text{with} \quad |\Lambda| > k \Rightarrow \mathcal{L}_\Lambda = 0 \ .$$
where $[N] := \{1,\dots,N\}$.
- $\max_{X\in\mathcal{L}_\Lambda} \sup_{t\geq 0} \|X_t\|_\infty$ bounded independent of N where $X \in \mathcal{L}$ means that X is one of the operators occurring in the Lindblad representation
$$\mathcal{L}_\Lambda = -\mathrm{i}[H_\Lambda, \cdot\,] + \sum_{\mu=1}^{d^k} \mathcal{D}[L_{\Lambda,\mu}]$$
with $\mathcal{D}[X](\rho) := 2X\rho X^\dagger - \{X^\dagger X, \rho\}$.
- $\mathcal{L}_\Lambda$ depend on time piecewise continuously.

Preliminaries

- Propagators $T_\mathcal{L}(t,s)$ defined by $\rho(t) = T_\mathcal{L}(t,s)\rho(s)$ for $t \geq s$ uniquely solve
$$\frac{\mathrm{d}}{\mathrm{d}t} T_\mathcal{L}(t,s) = \mathcal{L}_t T_\mathcal{L}(t,s) \ , \quad T_\mathcal{L}(s,s) = \mathrm{id} \ .$$
- Goal: Approximate the channel $T_\mathcal{L}(t,s)$ by local propagators $T_{\mathcal{L}_\Lambda}(t\frac{j}{m}, t\frac{j-1}{m})$.
- Corresponding situation in the case of time constant Hamiltonians, i.e. $\mathcal{L}_t = -\mathrm{i}[H_\Lambda, \cdot\,]$:
  - $T_\mathcal{L}(t,s)\rho = \mathrm{e}^{-\mathrm{i}H(t-s)}\rho\,\mathrm{e}^{\mathrm{i}H(t-s)}$,
  - $\mathrm{e}^{\mathrm{i}Ht} \approx (\prod_{\Lambda\subset[N]} \mathrm{e}^{\mathrm{i}H_\Lambda t/m})^m$ with error of order $\mathrm{O}(t^2N^{2k}/m)$ in ∞-norm.
- Physically relevant norm for states $\|\rho\|_1 := \mathrm{tr}(|\rho|)$ induces the $(1\to 1)$-norm $\|T\|_{1\to 1} := \sup_{\|A\|_1=1} \|T(A)\|_1$ for channels.

Main result

Theorem (Trotter decomposition of Liouvillian dynamics)

Let $\mathcal{L} = \sum_{\Lambda\subset[N]} \mathcal{L}_\Lambda$ be a k-local Liouvillian that acts on N subsystems with local Hilbert space dimension d. Furthermore, let $\mathcal{L}_\Lambda$ be piecewise continuous in time with the property that $a = \max_\Lambda \max_{X\in\mathcal{L}_\Lambda} \sup_{t\geq 0} \|X_t\|_\infty \in \mathrm{O}(1)$.
Then the error of the Trotter decomposition of a time evolution up to time t into m time steps is
$$\Big\| T_\mathcal{L}(t,0) - \prod_{j=1}^{m} \prod_{\Lambda\subset[N]} T_{\mathcal{L}_\Lambda}(t\tfrac{j}{m}, t\tfrac{j-1}{m}) \Big\|_{1\to 1} \leq \frac{cK^2t^2\mathrm{e}^{bt/m}}{m}, \tag{1}$$
where $c \in \mathrm{O}(d^{2k})$, $b \in \mathrm{O}(d^k)$, and $K \leq N^k$ is the number of strictly k-local terms $\mathcal{L}_\Lambda \neq 0$.
Furthermore, $T_{\mathcal{L}_\Lambda}(t\frac{j}{m}, t\frac{j-1}{m})$ can be replaced by the propagator $T_{\mathcal{L}_\Lambda^{\mathrm{av}}}(t\frac{j}{m}, t\frac{j-1}{m}) = \exp(t/m\mathcal{L}_\Lambda^{\mathrm{av}})$ of the average Liouvillian
$$\mathcal{L}_\Lambda^{\mathrm{av}} = \frac{m}{t}\int_{t(j-1)/m}^{tj/m} \mathcal{L}_\Lambda \mathrm{d}t$$
without changing the scaling (1) of the error.

Implications

Using Stinespring's dilation and the Solovay-Kitaev algorithm one obtains the following:

Implication (Dissipative Church-Turing theorem)

Time dependent Liouvillian dynamics can be simulated efficiently in the standard unitary circuit model.

The statement of [2] about Hilbert space being a "convenient illusion" can be lifted to mixed states and the physically relevant 1-norm:

Implication (Limitations of efficient state generation)

Let X_t^ρ be the set of states resulting from the time evolution of an arbitrary initial state ρ under all possible (time dependent) k-local Liouvillians up to some time t. For times t that are polynomial in the system size, the relative volume of X_t^ρ (measured in the operational metric induced by the 1-norm) is exponentially small.

Together with the locality of dissipative dynamics from [3] we can establish the following:

Implication (Simulation on classical computers [3])

For systems with short-range interactions and fixed time τ, the evolution of local observables can be simulated on classical computers with a system-size independent cost and arbitrary precision, e.g., using a variant of the time-dependent density matrix renormalization group method.

References

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