BosonSampling in the light of sample complexity: a review

C. Gogolin, M. Kliesch, L. Aolita, and J. Eisert

Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany

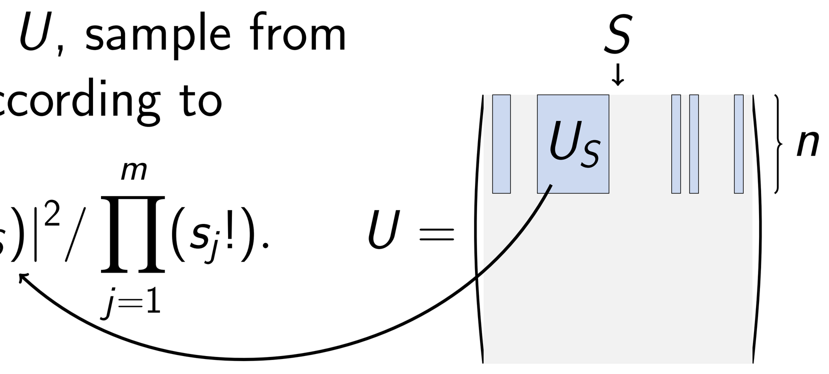
BosonSampling is a classically computationally hard problem that can — in principle — be efficiently solved with quantum linear optical networks. Recently, this has lead to an experimental race to implement such devices. With this poster we provide a review of the state of affairs concerning the possibility of certifying such devices.

Reference: arXiv:1306.3995.

BosonSampling

- Abstract problem:
  Given n and a fixed $m \times m$ unitary matrix U, sample from $\{S = (s_1, \dots, s_m) \colon s_j \geq 0 \wedge \sum_j s_j = n\}$ according to

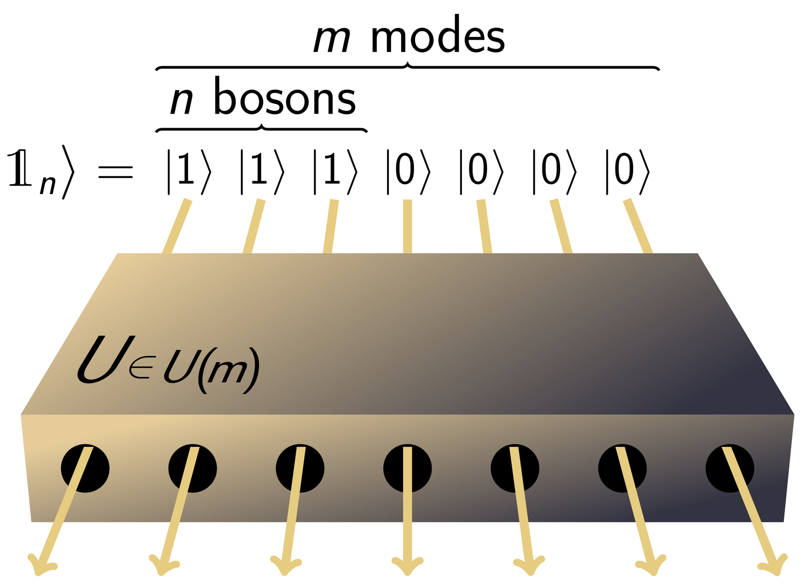
$$\Pr_{\mathcal{D}_U}[S] := |\operatorname{Perm}(U_S)|^2 / \prod_{j=1}^{m} (s_j!). \qquad U =$$



- Physical realization with quantum linear optical networks:
  Given n and a fixed $m \times m$ unitary matrix U, generate Fock states $|\mathbb{1}_n\rangle = |(1, \dots, 1, 0, \dots, 0)\rangle$, perform optical network corresponding to U (Hilbert space representation $\varphi(U)$) and measure in the Fock basis. Then the probability to get output sequence S is

$$|\langle 1_n|\varphi(U)|S\rangle|^2 = \Pr_{\mathcal{D}_U}[S].$$

Input: Fock state $|\mathbb{1}_n\rangle = |1\rangle\,|1\rangle\,|1\rangle\,|0\rangle\,|0\rangle\,|0\rangle\,|0\rangle$



U and n define an instance of BosonSampling.

Output: sample $S = (\ s_1\ ,\ s_2\ ,\ s_3\ ,\ s_4\ ,\ s_5\ ,\ s_6\ ,\ s_7\ )$

BosonSampling is hard...

- Even approximate BosonSampling is hard:

  > If 1-norm approximate classical efficient BosonSampling were possible for Haar random U and $m \in \Omega(n^5)$ then Ref. [1] provides strong evidence that the polynomial hierarchy would collapse to the third level.

  (Based on hardness of approximating permanents. Still holds after post selection on bit string outcomes.)
- Quantum linear optical networks are relatively easy to implement.
- ⇒ Recent attempts of physical realizations and loud claims: [3, 4, 5, 6, 8, 7]

...but not an NP problem.

- BosonSampling is a sampling problem, hence not in NP.
- Calculating $\Pr_{\mathcal{D}_U}[S]$ for fixed S from U is in general hard.
- ⇒ There is no obvious way to certify an implementation.

- ? How can BosonSampling devices be certified?
- ? What is the complexity of certification?
  First step: Look at complexity of distinguishing from uniform distribution.
- ? Can BosonSampling provide evidence against the Complexity-Theoretic Church–Turing Thesis?

What is certification of sampling experiments?

A BosonSampling certification algorithm receives samples and a description of the supposed BosonSampling instance (m, n, U) and must:

1. reject with prob $\geq 2/3$ if distribution is not 1-norm close to $\mathcal{D}_U$.
2. accept with prob $\geq 2/3$ if distribution is $\mathcal{D}_U$.

References

[1] S. Aaronson and A. Arkhipov, Proc. of STOC (2011).
[2] S. Aaronson and A. Arkhipov, ECCC, TR13-135 (2013).
[3] J. B. Spring, et al., Science 339, 798 (2013).
[4] M. Tillmann, et al.,Nature Photonics, 7, 540–544 (2013).
[5] M. A. Broome, et al.,Science 339, 794 (2013).
[6] A. Crespi, et al.,Nature Photonics 7, 545549 (2013).
[7] N. Spagnolo et al.,arXiv:1311.1622.
[8] J. Carolan, et al.,arXiv: 1311.2913.
[9] C. Gogolin, M. Kliesch, L. Aolita, and J. Eisert, arXiv:1306.3995 (2013).
[10] M. C. Tichy, M. Tiersch, F. Mintert, and A. Buchleitner, New J. Phys. 14 093015 (2012).
[11] C. Gogolin, M. Kliesch, L. Aolita, and J. Eisert, in preperation.
[12] L. Trevisan, M. Tulsiani, and S. Vadhan, ECCC TR08-103 (2009).

Complexity of distinguishing $\mathcal{D}_U$ from uniform distribution [9]

1. State discrimination:
   - Assume $\mathcal{D}_U$ is completely known.
   - Note that this is unrealistic as approximating the $\Pr_{\mathcal{D}_U}[S]$ is computationally hard!
   - ⇒

     > Let $m \in O(\operatorname{poly}(n))$, then any instance of BosonSampling that could be potentially hard to sample 1-norm approximately classically can be distinguished from the uniform distribution from $O(n^{2+\epsilon})$ samples. ☺

2. Black box setting:
   - It is unclear how/whether U can be used in a computationally efficient way for certification.
   - ? What can we do without using U?
   - ⇒

     > If U Haar random and $m \in \Omega(n^\nu)$ with $\nu > 3$ ($\nu > 2$ with post selection), then with probability supra-exponentially close to one in n no symmetric probabilistic algorithm can distinguish $\mathcal{D}_U$ from the uniform distribution from fewer than $\Omega(\mathrm{e}^{n/2})$ many samples ☹ .

     (Symmetric algorithms are invariant under permuting the sample space, i.e., only look at relative frequencies.)

Corroboration/partial certification

One can look at:

1. Boson bunching [10, 9]
2. Moments [11]
3. Row norm estimators [2]

   ...

- ⇒ Allows to efficiently distinguish from uniform distribution.
- ⇒ But: This yields no certification. All algorithms can be fooled.

Obstacle for efficient classical certification

- $\mathcal{D}_U$ typically has a high min-entropy [9].
- ⇒ F. Brandao (published in [2] based on [12]):

  > For every instance of BosonSampling with high min-entropy and every circuit length $T \in O(\operatorname{poly}(n))$ there is a classically efficiently samplable distribution indistinguishable from $\mathcal{D}_U$ by all circuits of length T.

- ⇒ It can only be certified that a supposed BosonSampling black box has polynomially more computational power than the computation that was used to check its validity.
- ⇒ Challenges usefulness of BosonSampling experiments to provide evidence against the Complexity-Theoretic Church–Turing Thesis.