Under what conditions do quantum systems thermalize?

New insights from quantum information theory

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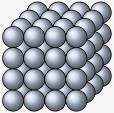
University of Potsdam

DPG March Meeting Dresden

Irreversibility from quantum many body dynamics



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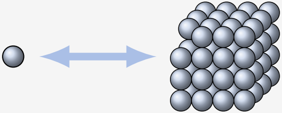

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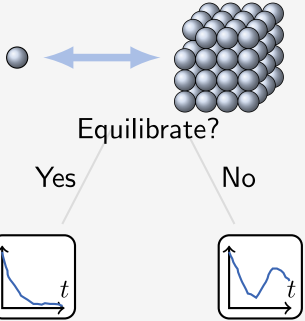

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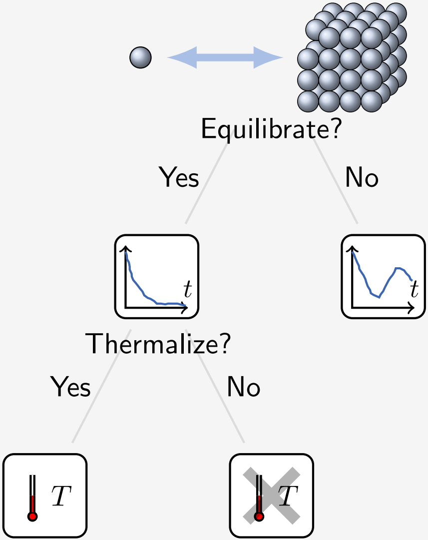

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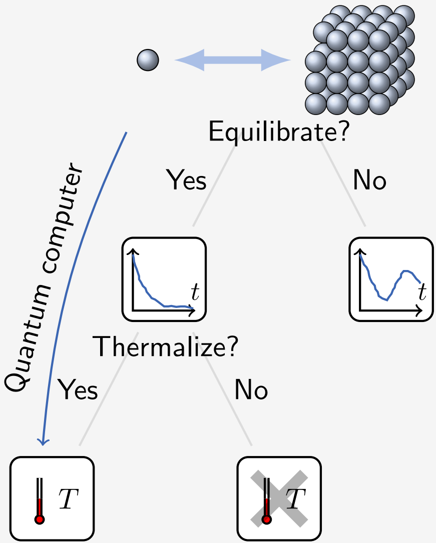
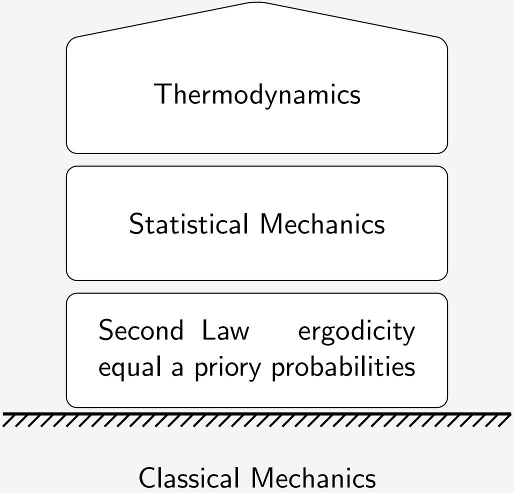
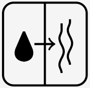


New foundation for statistical mechanics



New foundation for statistical mechanics

Thermodynamics



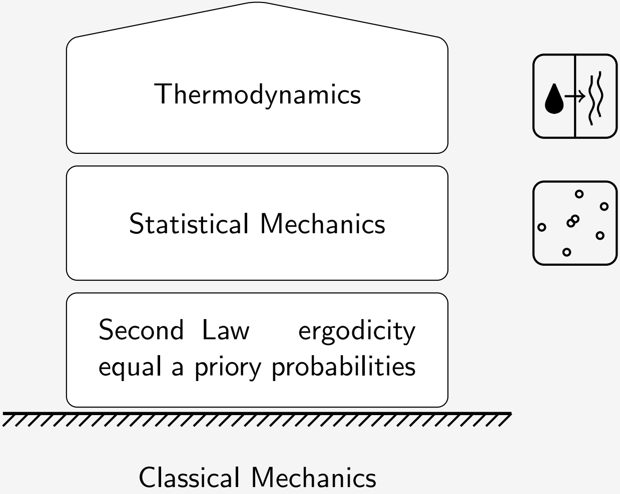
Statistical Mechanics

Second Law ergodicity

equal a priory probabilities

Classical Mechanics

New foundation for statistical mechanics



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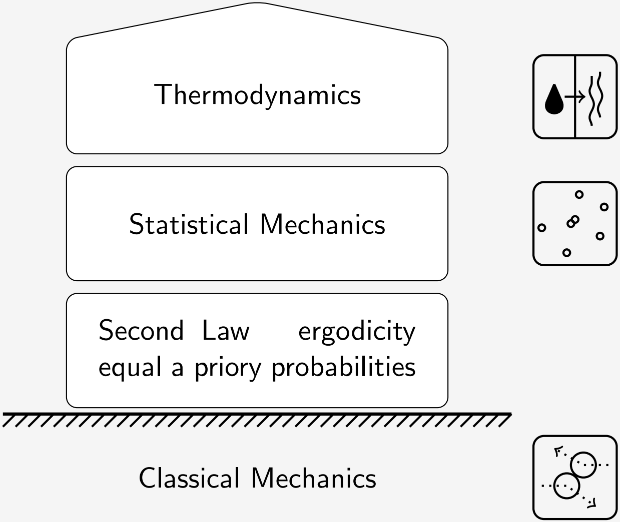

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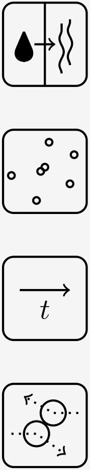

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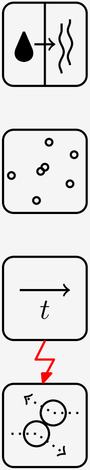
Second Law ergodicity
equal a priory probabilities ?

Classical Mechanics ?



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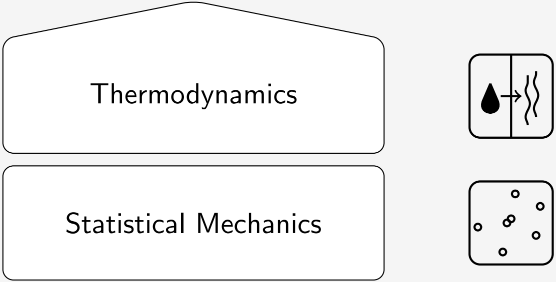

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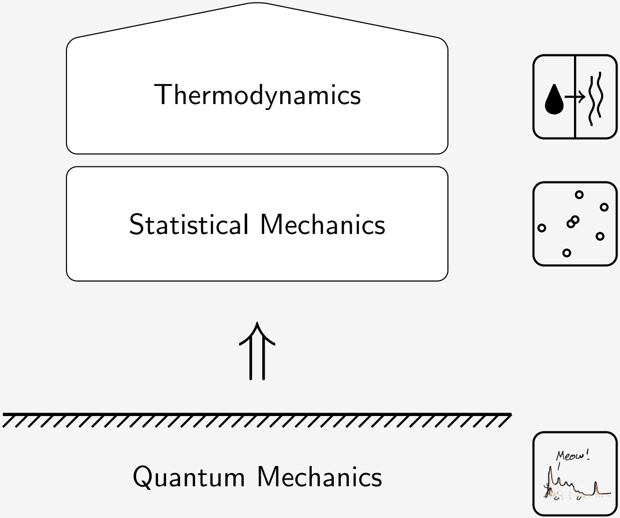
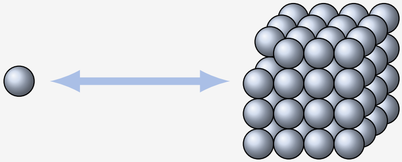


Thermalization is a complicated process



Thermalization implies:

1. Equilibration [1, 2, 3]
2. Subsystem initial state independence [4]
3. Weak bath state dependence [5]
4. Diagonal form of the subsystem equilibrium state [6]
5. Gibbs state $\mathrm{e}^{-\beta\,\mathscr{H}}$ [3, 5]

[1] P. Reimann, PRL 101 (2008) 190403
[2] N. Linden, S. Popescu, A. J. Short, and A. Winter, PRE 79 (2009) no. 6, 061103
[3] J. Gemmer, M. Michel, and G. Mahler, Springer (2009)
[4] C. Gogolin, M. P. Mueller, and J. Eisert, PRL 106 (2011) 040401
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[6] C. Gogolin, PRE 81 (2010) no. 5, 051127

Thermalization and quantum integrability

There is a common belief in the literature [7, 8, 9, 10, 11] . . .

Non-integrable	$\Longrightarrow$	Thermalization
Integrable	$\Longrightarrow$	No thermalization

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[8] S. Manmana, S. Wessel, R. Noack, and A. Muramatsu, ibid. 98 (2007) 210405
[9] M. Rigol, V. Dunjko, and M. Olshanii, Nature 452 (2008) 854
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Thermalization and quantum integrability

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...which is unfortunately not quite true.

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Absence of thermalization in non integrable systems

Result (Theorem 1 and 2 in [4]):

- Too little (geometric) entanglement in the energy eigenbasis prevents initial state independence.
- This can happen even in non-integrable systems.

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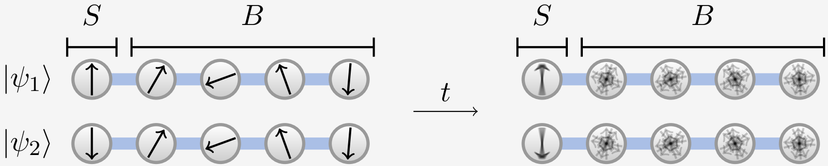

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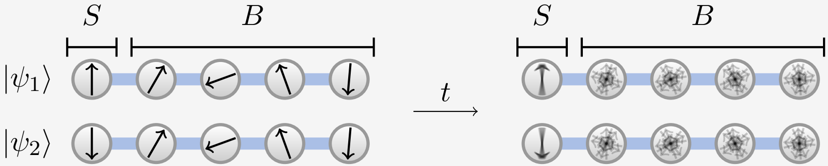

$$\mathcal{D}(\uparrow, \downarrow) \geq \mathcal{D}(\uparrow, \downarrow) - R(|\psi_1\rangle) - R(|\psi_2\rangle).$$

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Result (Theorem 1 and 2 in [4]):

Corollary of Theorem 2 in [4]: Effective entanglement in the eigenbasis (for spin 1/2)

Let $\{|i\rangle\}$ be a basis for S, then if

$$\delta = \max_k \min_i \mathcal{D}(\mathrm{Tr}_B \, |E_k\rangle\langle E_k|, |i\rangle\langle i|)$$

is small, then for all $|i\rangle$ and almost all $|\psi^B\rangle$

$$R(|i\rangle \otimes |\psi^B\rangle) \leq 4\,\delta.$$

$|\psi_1\rangle$

$|\psi_2\rangle$

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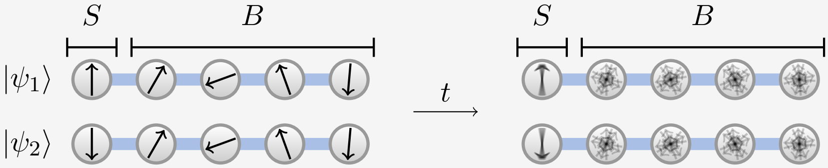

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Conclusions and outlook

We have seen in this talk:

$$\text{Non-integrability} \not\Longrightarrow \text{Thermalization}$$

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[13] M. Cramer and J. Eisert, NJP 12 (2010) 055020

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But there is more:

- Rigorous results on Equilibration [1, 2]
- A strong connection to decoherence [6]
- A quantum algorithm to prepare Gibbs states [5].
- Thermalization in exactly solvable models [12, 13]

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The major open question:

- Time scales. How long does it take to equilibrate?

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Thank you for your attention!

⟶ slides: www.cgogolin.de

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